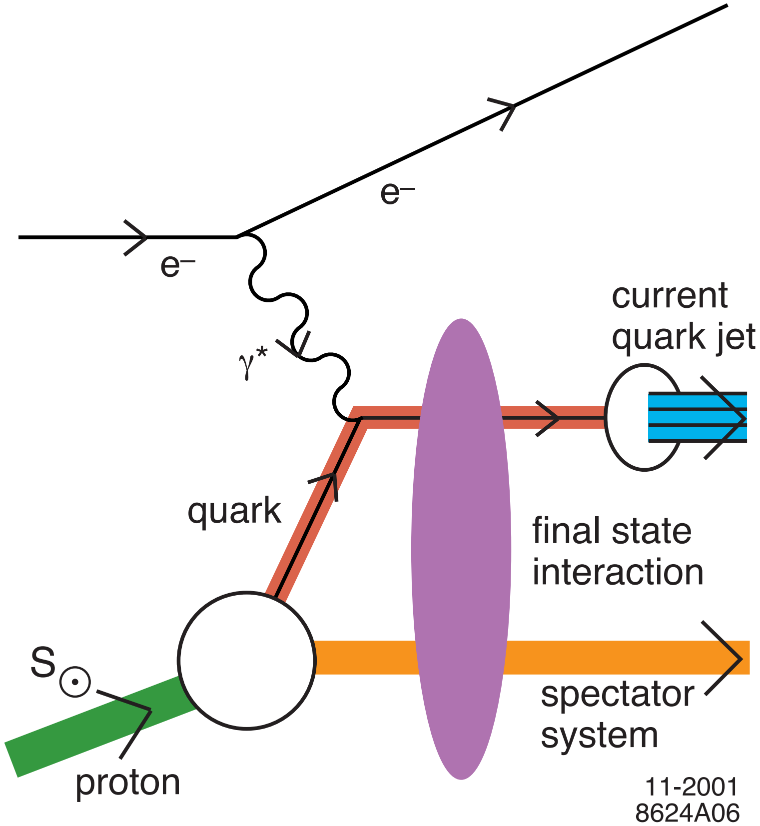
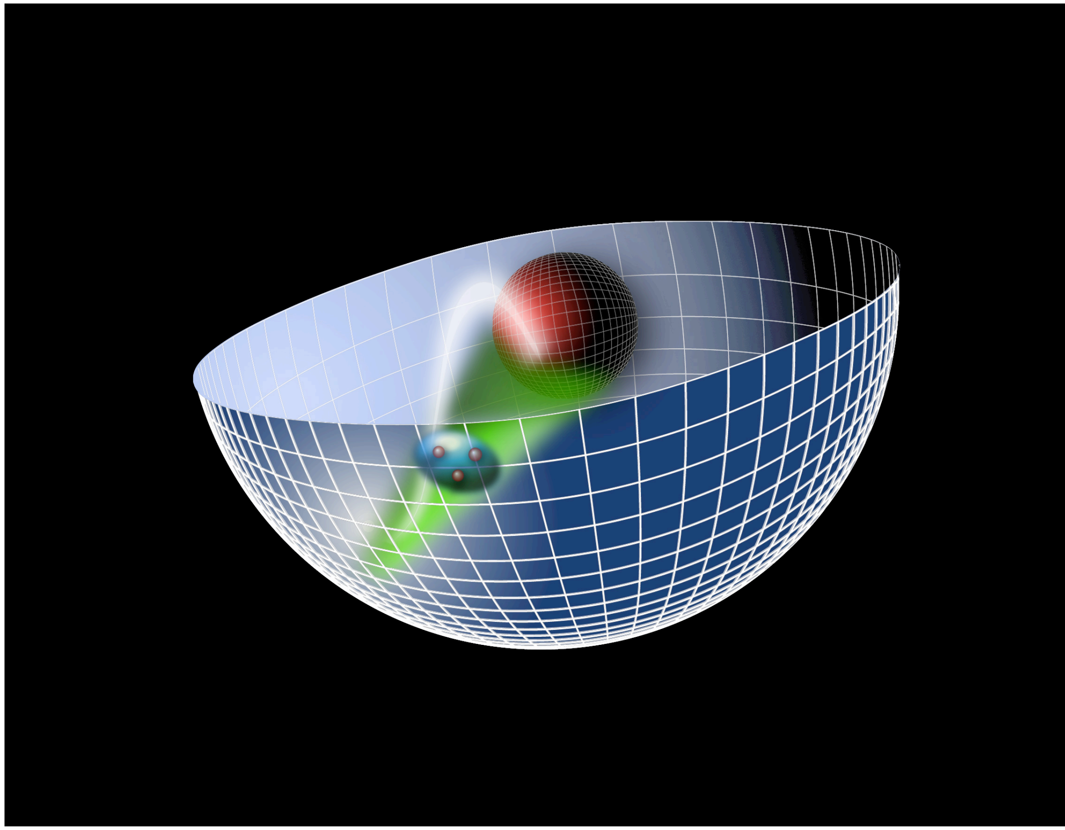
Novel QCD Spin Phenomenology and Light Front Holography



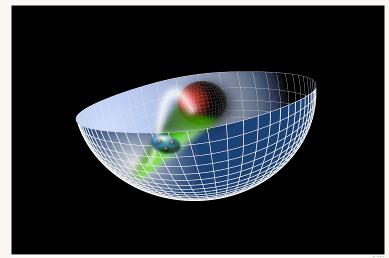


Stan Brodsky, SLAC

LBNL Spin Workshop

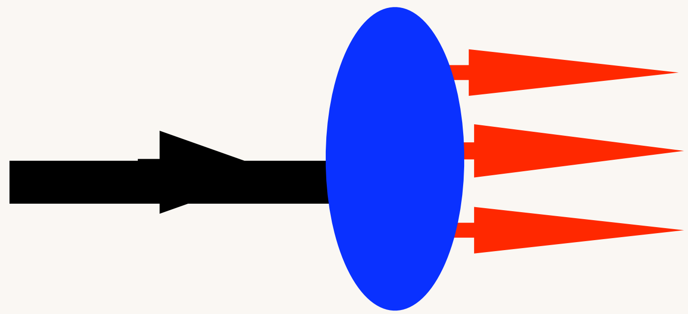
June 5, 2009

- *Angular Momentum and Spin Phenomena in QCD*
- *Essentials of Spin on the Light Front*
- New Insights from higher space-time dimensions: *AdS/QCD*



- *Light-Front Holography*
- *Light Front Wavefunctions:* analogous to the Schrodinger wavefunctions of atomic physics

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



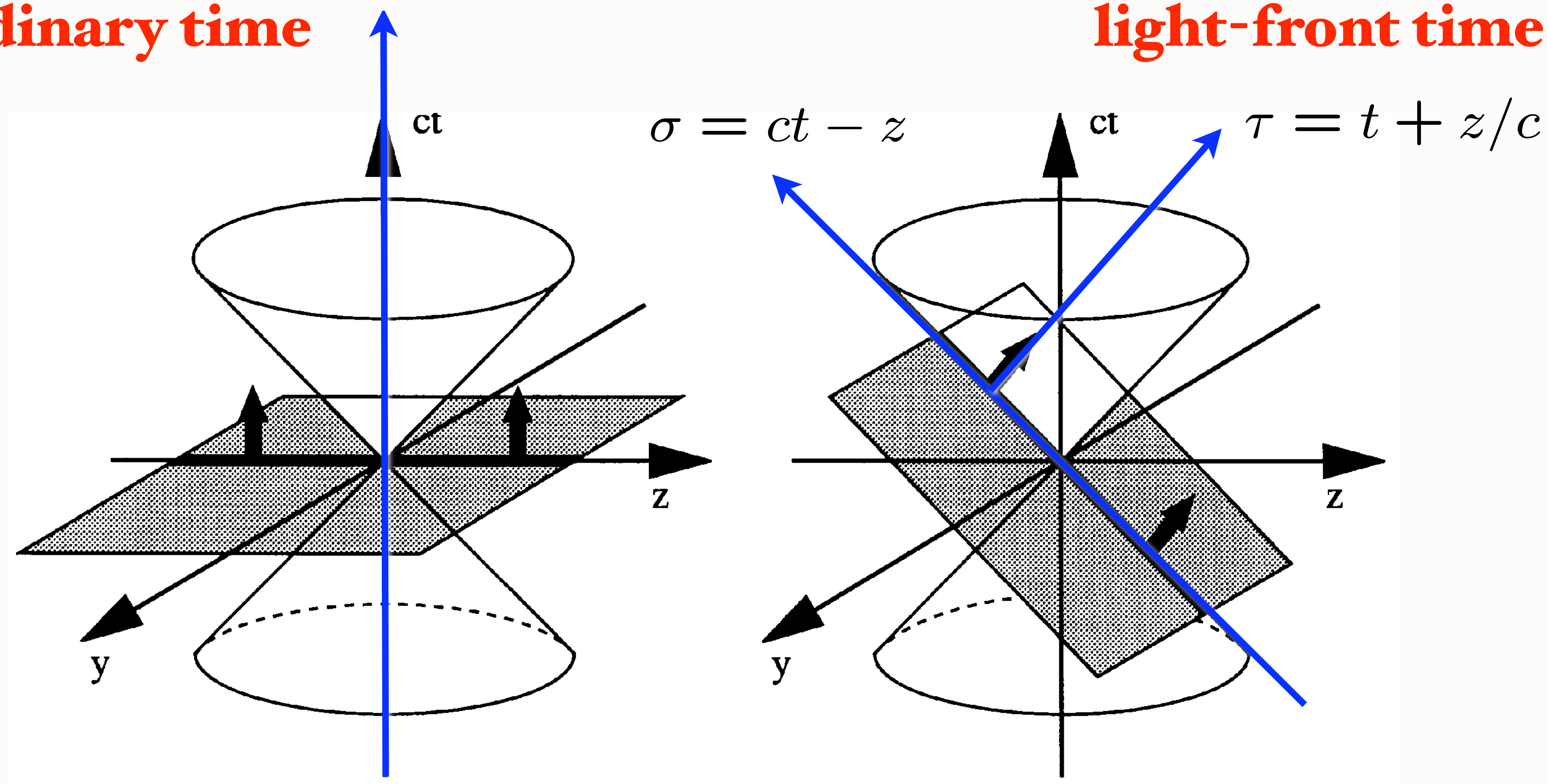
- *Hadronization at the Amplitude Level*

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!



Instant Form **Front Form**

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

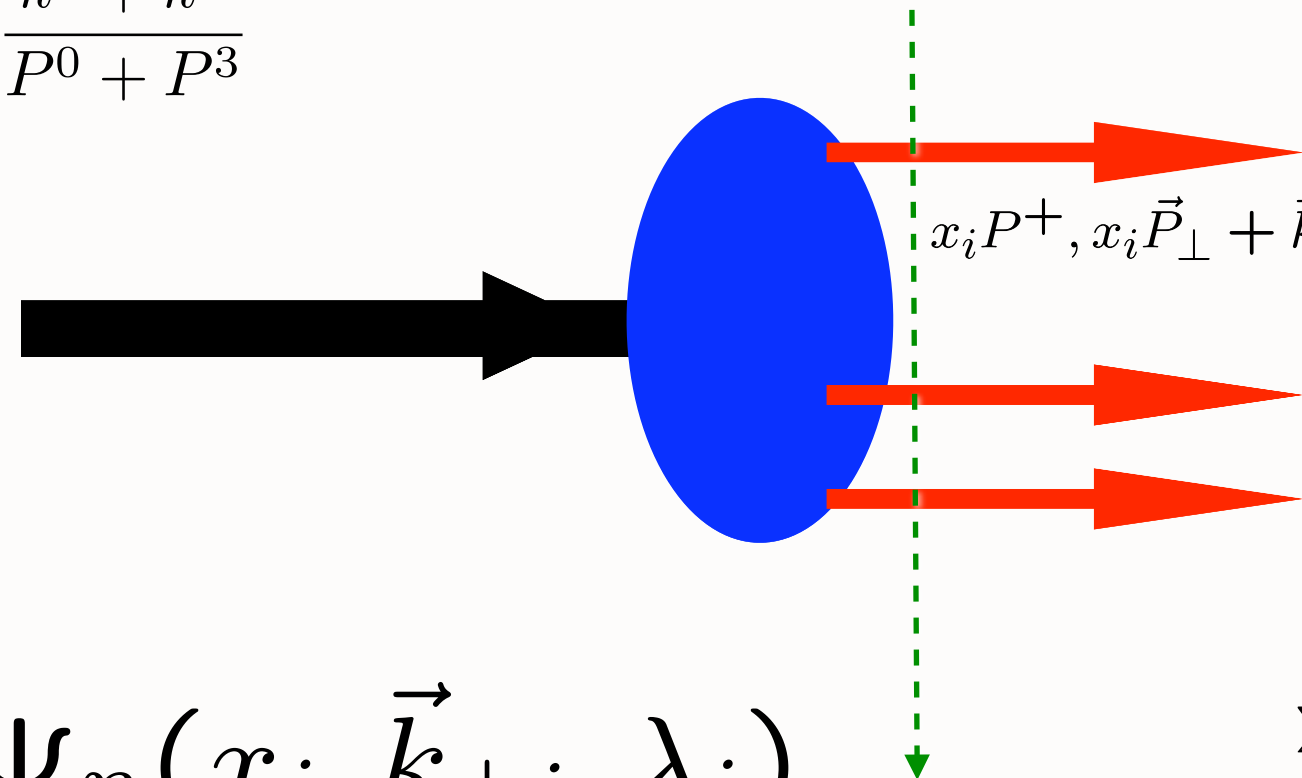
Causally-Connected Domains

HELEN BRADLEY - PHOTOGRAPHY

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$P^+, \vec{P}_\perp$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State!

LF Spin Sum Rule

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

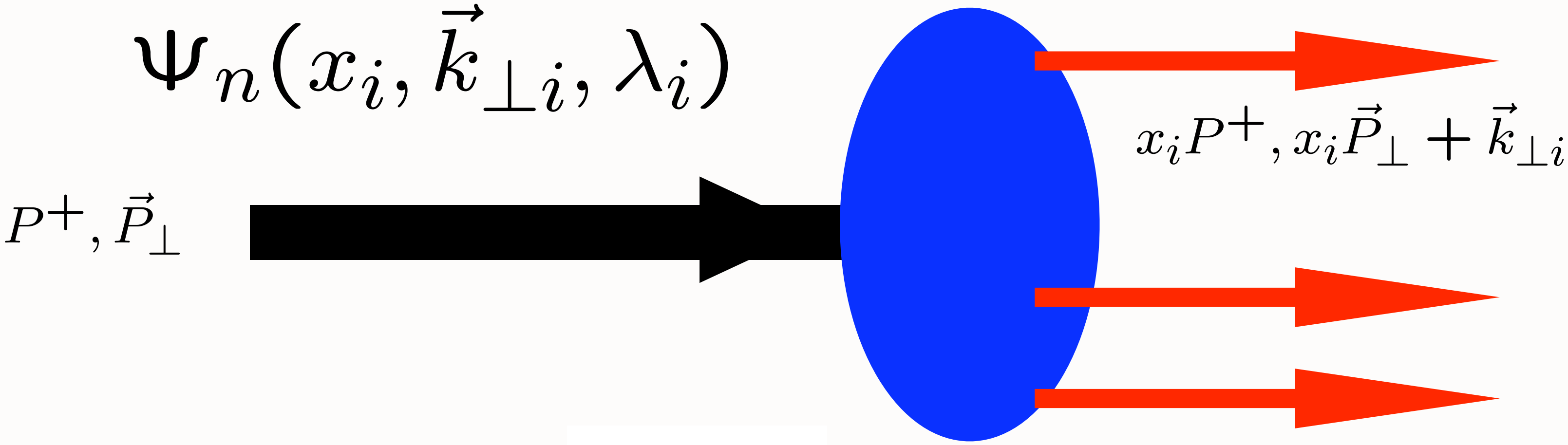
n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

$\sum_i^n x_i = 1$ $\qquad$ $\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$\sum_{i=1}^n k_i^+ = \sum_{i=1}^n x_i \vec{P}^+ = \vec{P}^+$ $\qquad$ $\sum_{i=1}^n (x_i \vec{P}_\perp + \vec{k}_{\perp i}) = \vec{P}_\perp$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right) \qquad j = 1, 2, \cdots (n-1)$$

n-1 Intrinsic Orbital Angular Momenta

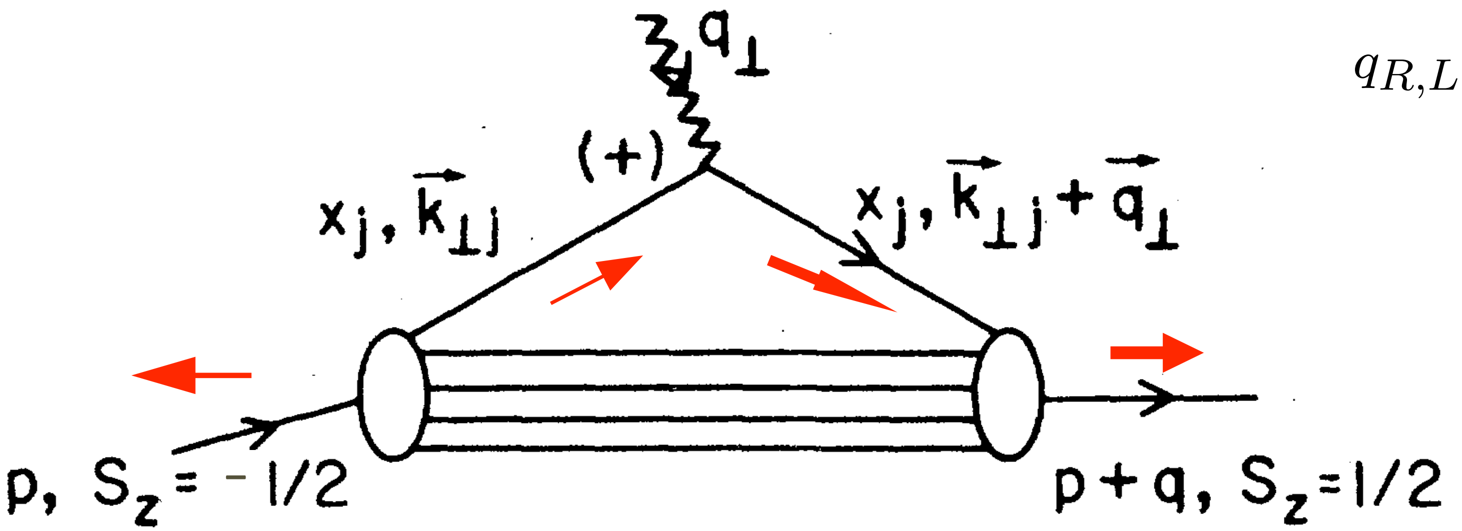
Frame Independent

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\,\psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\,\psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



$$q_{R,L} = q^x \pm i q^y$$

$\boxed{\text{Must have } \Delta\ell_z = \pm 1 \text{ to have nonzero } F_2(q^2)}$

Same matrix elements appear in Sivers effect
-- connection to quark anomalous moments

Anomalous gravitomagnetic moment B(0)

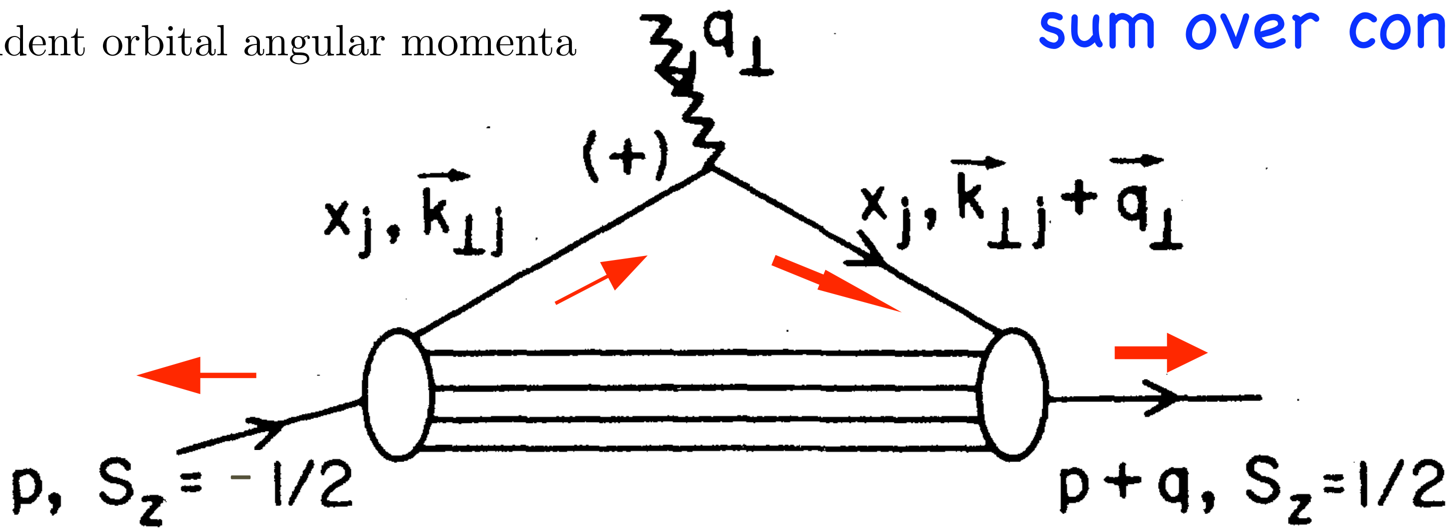
Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*

graviton

$\sum_{i=1}^{n} L_i = 0$

$n - 1$ independent orbital angular momenta

sum over constituents

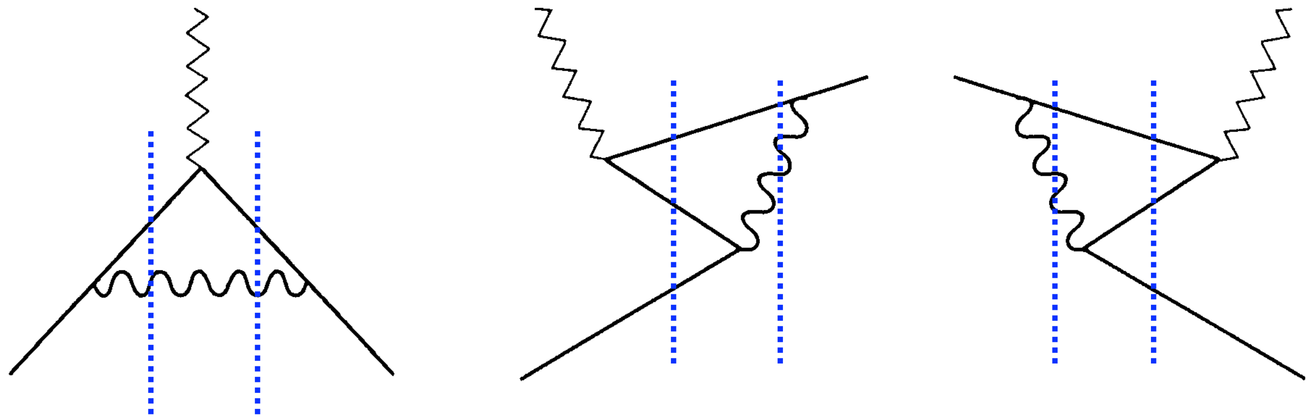


Hwang, Schmidt, sjb;
Holstein et al

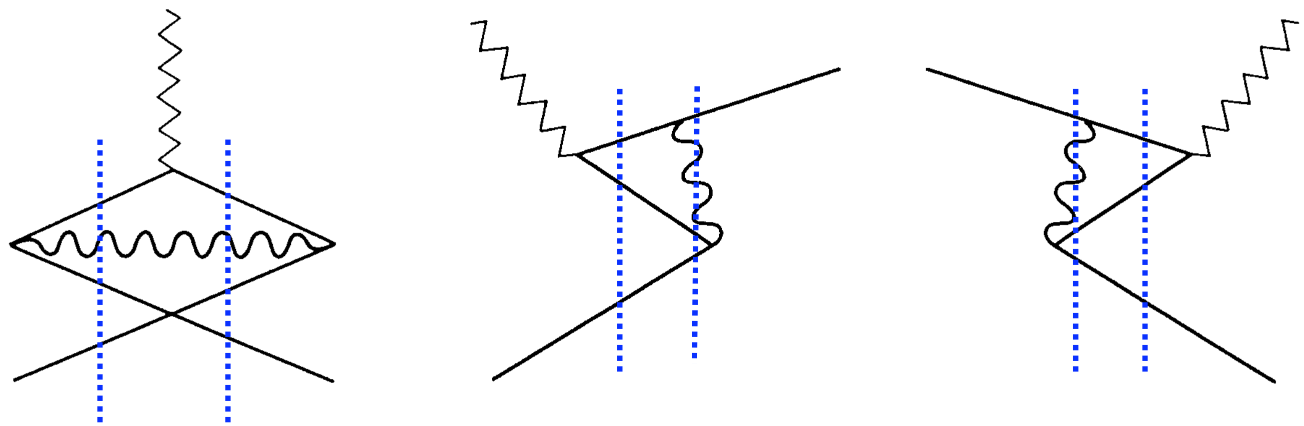
$\boxed{B(0) = 0}$

Each Fock State

Calculation of lepton g-2 in TOPTH (Instant form)



Only diagram in LFPTH (q^+=0)



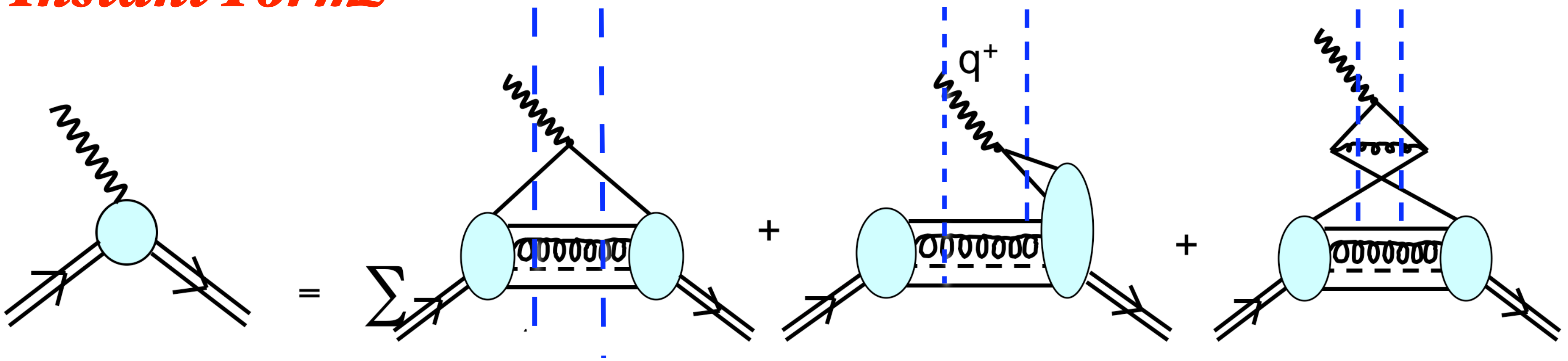
n! diagrams at order e^n

energy denominators:
frame-dependent and non-analytic

$$\sqrt{(\vec{p}+\vec{q}-\vec{k})^2+m^2}$$

Calculation of Form Factors in Equal-Time Theory

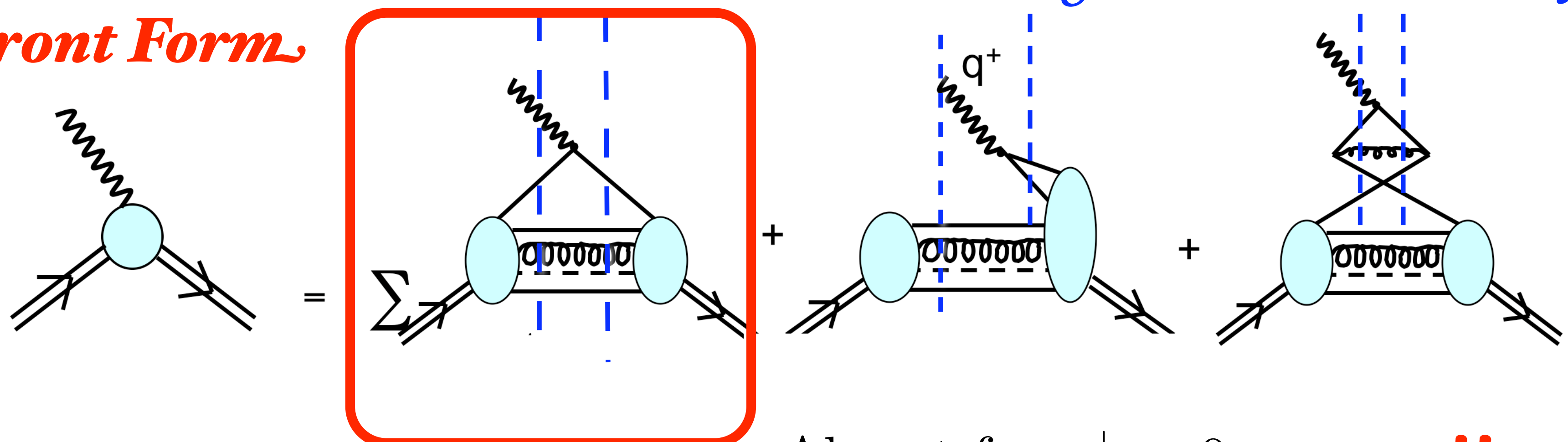
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

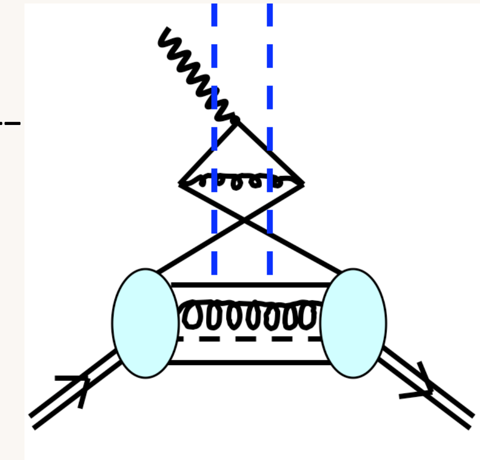
Front Form



Absent for $q^+ = 0$ ***zero !!***

Calculation of Hadron Form Factors Instant Form

- Current matrix elements of hadron include interactions with vacuum-induced currents arising from infinitely-complex vacuum
- Pair creation from vacuum occurs at any time before probe acts -- acausal
- Knowledge of hadron wavefunction insufficient to compute current matrix elements
- Requires dynamical boost of hadron wavefunction -- unknown except at weak binding
- Complex vacuum even for QED
- None of these complications occur for quantization at fixed LF time (front form)



Special Features of LF Spin

- LF Helicity and chirality refer to z direction, not the particle's 3-momentum p
- LF spinors are eigenstates of $S^z = \pm\frac{1}{2}$
- Gluon polarization vectors are eigenstates of $S^z = \pm 1$

$$\epsilon^\mu = (\epsilon^+, \epsilon^-, \vec{\epsilon}_\perp) = (0, 2\frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k^+}, \vec{\epsilon}_\perp)$$

$$\vec{\epsilon}^{\,\pm}_\perp = \mp\frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y}) \qquad k^\mu \epsilon_\mu = 0$$

$$\left.\begin{matrix} u_\uparrow(p) \\ u_\downarrow(p) \end{matrix}\right\} = \frac{1}{(p^+)^{1/2}} \left(p^+ + \beta m + \alpha_\perp \cdot \mathrm{p}_\perp\right) \times \left\{\begin{matrix} \chi(\uparrow) \\ \chi(\downarrow) \end{matrix}\right. ,$$

$$\left.\begin{matrix} v_\uparrow(p) \\ v_\downarrow(p) \end{matrix}\right\} = \frac{1}{(p^+)^{1/2}} \left(p^+ - \beta m + \vec{\alpha}_\perp \cdot \vec{\mathrm{p}}_\perp\right) \times \left\{\begin{matrix} \chi(\downarrow) \\ \chi(\uparrow) \end{matrix}\right.$$

$$\chi(\uparrow) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi(\downarrow) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix},$$

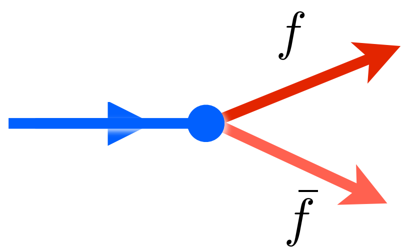
Matrix element $\bar{v}_{\lambda'} \cdots u_{\lambda}$	Helicity $(\lambda \to \lambda')$ $\uparrow \to \uparrow$ / $\downarrow \to \downarrow$	$\uparrow \to \downarrow$ / $\downarrow \to \uparrow$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma^+ \frac{u(q)}{(q^+)^{1/2}}$	0	2
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma^- \frac{u(q)}{(q^+)^{1/2}}$	$\mp \frac{2m}{p^+q^+}[(p^1 \pm ip^2) + (q^1 \pm iq^2)]$	$\frac{2}{p^+q^+}(p_\perp \cdot q_\perp \pm ip_\perp \times q_\perp - m^2)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma_\perp^i \frac{u(q)}{(q^+)^{1/2}}$	$\mp m \left(\frac{p^+ + q^+}{p^+q^+}\right)(\delta^{il} \pm i\delta^{i2})$	$\frac{p_\perp^i \mp i\epsilon^{ij} p_\perp^j}{p^+} + \frac{q_\perp^i \pm i\epsilon^{ij} q_\perp^j}{q^+}$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \frac{u(q)}{(q^+)^{1/2}}$	$\mp \left(\frac{p^1 \pm ip^2}{p^+} - \frac{q^1 \pm iq^2}{q^+}\right)$	$m\left(\frac{p^+ - q^+}{p^+q^+}\right)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma^- \gamma^+ \gamma^- \frac{u(q)}{(q^+)^{1/2}}$	$\mp \frac{8m}{p^+q^+}[(p^1 \pm ip^2) + (q^1 \pm iq^2)]$	$\frac{8}{p^+q^+}(p_\perp \cdot q_\perp \pm ip_\perp \times q_\perp - m^2)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma^- \gamma^+ \gamma_\perp^i \frac{u(q)}{(q^+)^{1/2}}$	$\mp \frac{4m}{p^+}(\delta^{il} \pm i\delta^{i2})$	$4\left(\frac{p_\perp^i \mp i\epsilon^{ij} p_\perp^j}{p^+}\right)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma_\perp^i \gamma^+ \gamma^- \frac{u(q)}{(q^+)^{1/2}}$	$\mp \frac{4m}{q^+}(\delta^{il} \pm i\delta^{i2})$	$4\left(\frac{q_\perp^i \pm i\epsilon^{ij} q_\perp^j}{q^+}\right)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}} \gamma_\perp^i \gamma^+ \gamma_\perp^j \frac{u(q)}{(q^+)^{1/2}}$	0	$2(\delta^{ij} \pm i\epsilon^{ij})$

G. P. Lepage and sjb

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} S_i^z + \sum_{i=1}^{n-1} L_i^z \qquad L_j^z = -i\left(k_j^x \frac{\partial}{\partial k_j^y} - k_j^y \frac{\partial}{\partial k_j^x}\right)$$

$z_3, \vec{k}_{\perp 3}$

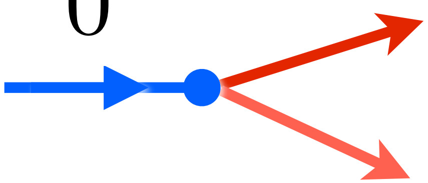


$z_2, \vec{k}_{\perp 2}$

$z_1, \vec{k}_{\perp 1}$

Spin-0 coupling to fermion pair

0



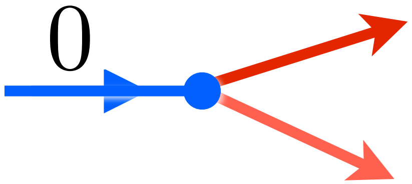
$+\frac{1}{2}$

$+\frac{1}{2}$

$$L^z = -1 \qquad < ij >= -\sqrt{2 z_i z_j} \vec{\epsilon}_\perp^{\,+} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

spinor overlap

0



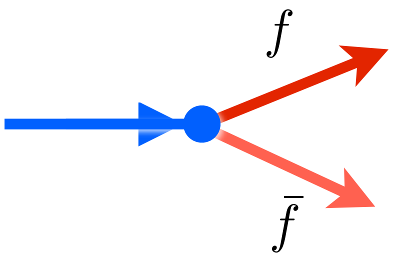
$-\frac{1}{2}$

$-\frac{1}{2}$

$$L^z = +1 \qquad [ij] = \sqrt{2 z_i z_j} \vec{\epsilon}^{\,(-)} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

Angular Momentum on the Light-Front

$z_3, \vec{k}_{\perp 3}$



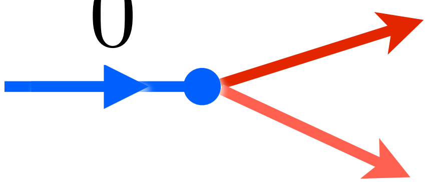
$z_2, \vec{k}_{\perp 2}$

$z_1, \vec{k}_{\perp 1}$

P-Wave Decay
Spin-0 coupling
to fermion pair

$L^z = -1$

0



$+\frac{1}{2}$

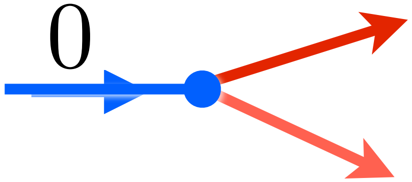
$+\frac{1}{2}$

spinor overlap

$$< ij > = < i - | j + > = -\sqrt{2 z_i z_j} \vec{\epsilon}_\perp^{\,+} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

$L^z = +1$

0



$-\frac{1}{2}$

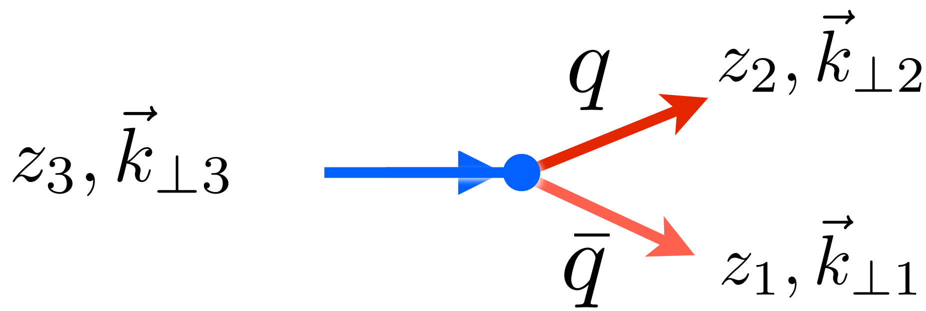
$-\frac{1}{2}$

$$[ij] = < i^+ | j - > = \sqrt{2 z_i z_j} \vec{\epsilon}^{(-)} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$

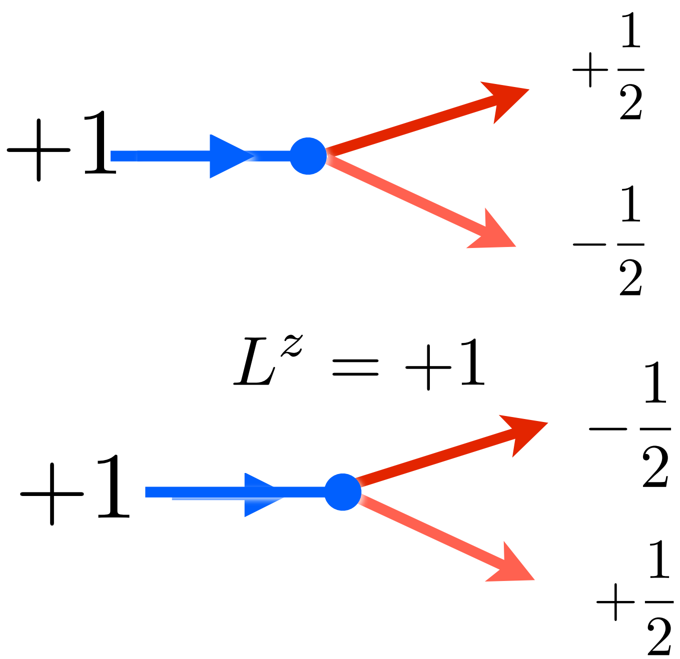
$$\boxed{< ij > [ij] = z_i z_j \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)^2 = \mathcal{M}^2_{ij}}$$

Identity

Angular Momentum on the Light-Front



Spin-1 coupling
to massless fermion pair



$$\vec{\epsilon}_\perp^{\,(+)} \cdot \frac{\vec{k}_{\perp 2}}{z_2} - \vec{\epsilon}_\perp^{\,(-)} \cdot \frac{\vec{k}_{\perp 3}}{z_3},$$

P-Wave Decay

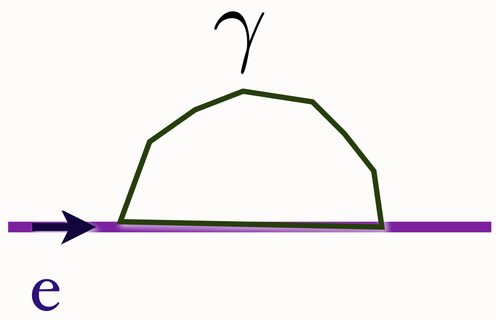
$$\vec{\epsilon}_\perp^{\,(-)} \cdot \frac{\vec{k}_{\perp 2}}{z_2} - \vec{\epsilon}_\perp^{\,(+)} \cdot \frac{\vec{k}_{\perp 3}}{z_3}$$

Compare CM distribution $1 + \cos^2 \theta_{CM}$

Looks like S and D-Wave Decay

Orbital angular momentum of electron carried by photon at LO in QED

$$< L^z >_{\Lambda^2} = -\frac{\alpha}{4\pi}[\frac{4}{3}\log\frac{\Lambda^2}{m^2} - \frac{2}{9}]$$



$$\frac{d}{d\log Q^2} < L^z >_{Q^2} = -\frac{\alpha}{3\pi}$$

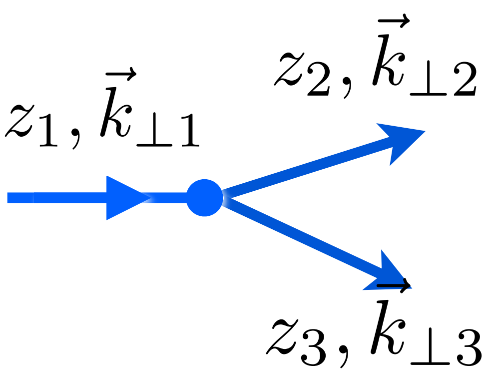
Evolution of OAM

Angular Momentum Decomposition for an Electron.
Matthias Burkardt, Hikmat BC (New Mexico State U.) . JLAB-THY-08-920, Dec 2008. 7pp.
e-Print: **arXiv:0812.1605** [hep-ph]

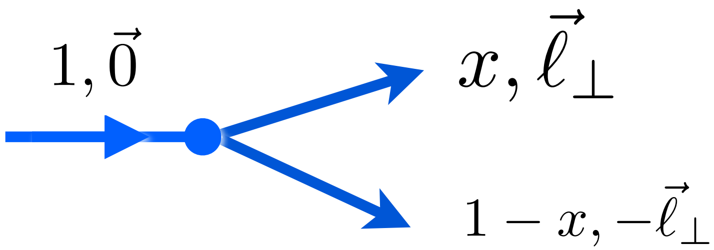
Light cone representation of the spin and orbital angular momentum of relativistic composite systems.
Stanley J. Brodsky (SLAC) , Dae Sung Hwang (Sejong U.) , Bo-Qiang Ma (CCAST World Lab, Beijing & Peking U. & Beijing, Inst. High Energy Phys.) , Ivan Schmidt (Santa Maria U., Valparaiso) . SLAC-PUB-8392, USM-TH-90, Mar 2000. 28pp.
Published in **Nucl.Phys.B593:311-335,2001**.
e-Print: **hep-th/0003082**

Angular Momentum on the Light-Front

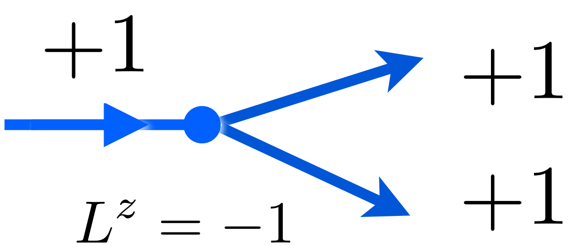
Triple-Gluon Coupling



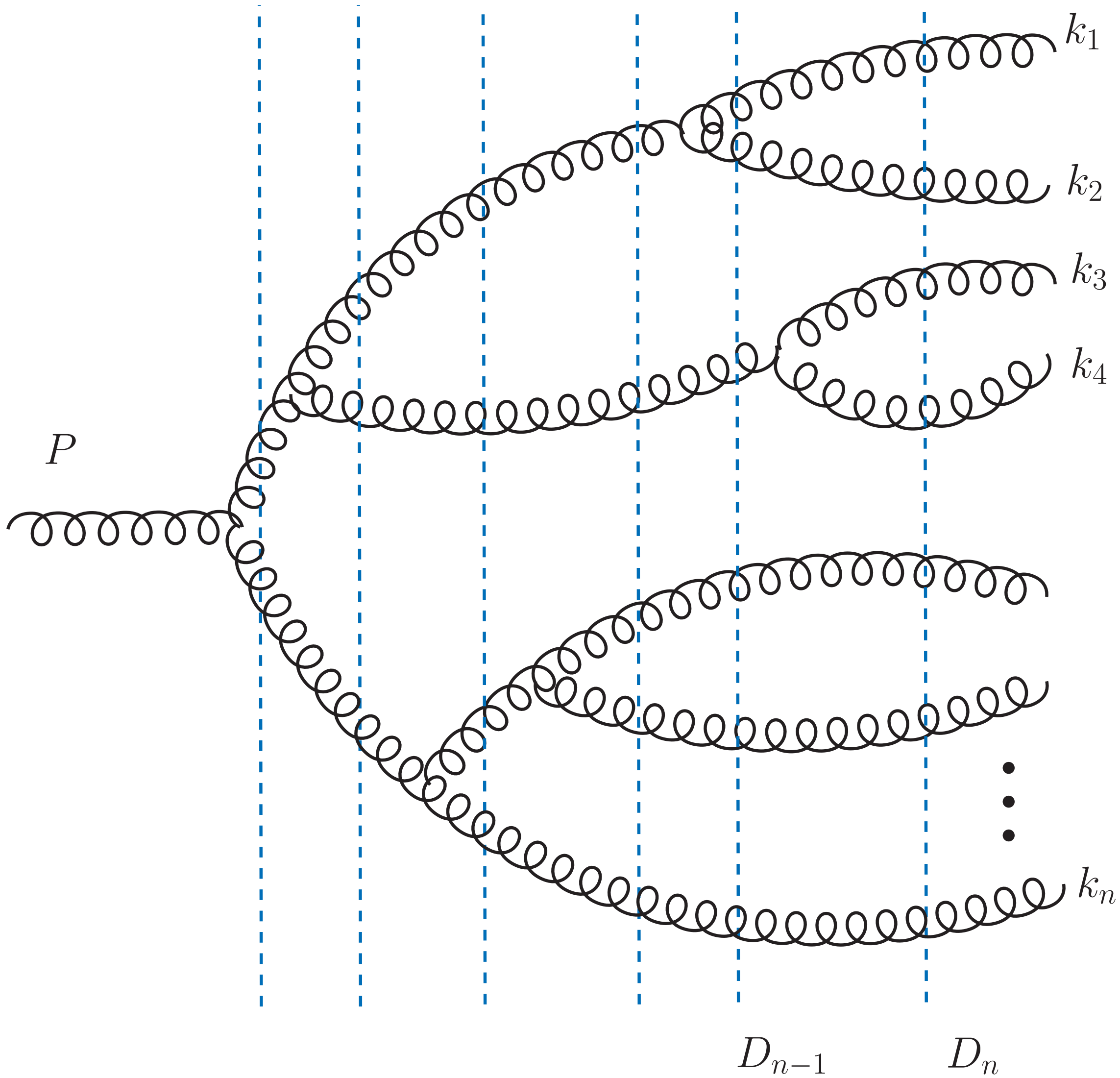
$$g z_1 \vec{\epsilon}_\perp^{\,+} \cdot \vec{v}_{23} = g z_1 \vec{\epsilon}_\perp^{\,+} \cdot \left(\frac{\vec{k}_{\perp 2}}{z_2} - \frac{\vec{k}_{\perp 3}}{z_3}\right)$$



$$g z_1 \vec{\epsilon}_\perp^{\,+} \cdot \vec{v}_{23} = g \vec{\epsilon}_\perp^{\,+} \cdot \frac{\vec{\ell}_\perp}{x(1-x)}$$



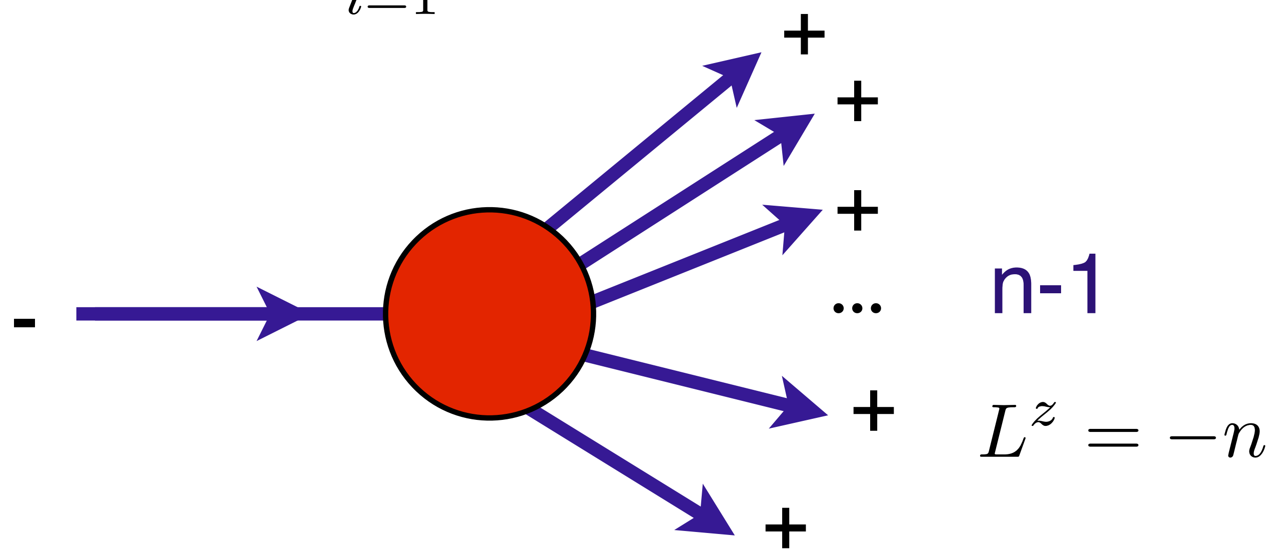
$$< ij > = -\sqrt{2 z_i z_j} \vec{\epsilon}_\perp^{\,+} \cdot \left(\frac{\vec{k}_{\perp i}}{z_i} - \frac{\vec{k}_{\perp j}}{z_j}\right)$$



Exact kinematics in the small x evolution of the color dipole and gluon cascade.
Leszek Motyka (Hamburg U. & Jagiellonian U.) , Anna M. Stasto (Penn State U. & RIKEN BNL & Cracow, INP) . Jan 2009. 37pp.
e-Print: **arXiv:0901.4949** [hep-ph]

$$M(-1 \to +1 + 1 \cdots + 1) \propto g^{n-2} = 0$$

$$J^z = -1 = \sum_{i=1}^{n} S_i^z + L^z = (n-1) + L^z$$

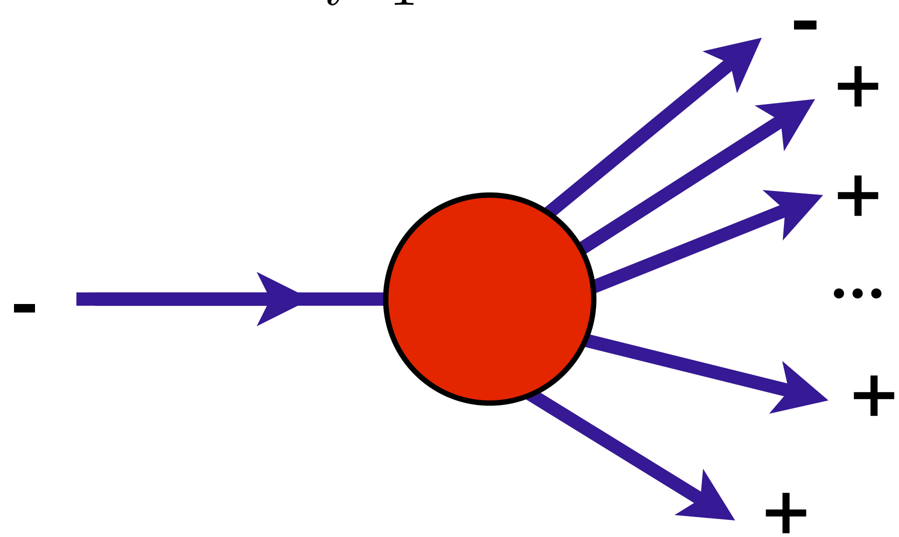


Vanishes Because Maximum $|L^z| = n - 2$

Renormalizability

$$M(-1 \to -1+1+1+1 \cdots +1) \propto g^{n-2} = 0$$

$$J^z = -1 = \sum_{i=1}^{n} S_i^z + L^z = (n-2) + L^z$$

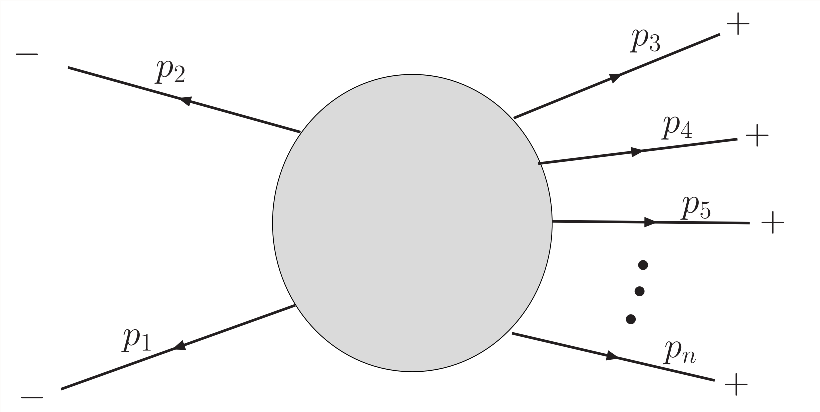


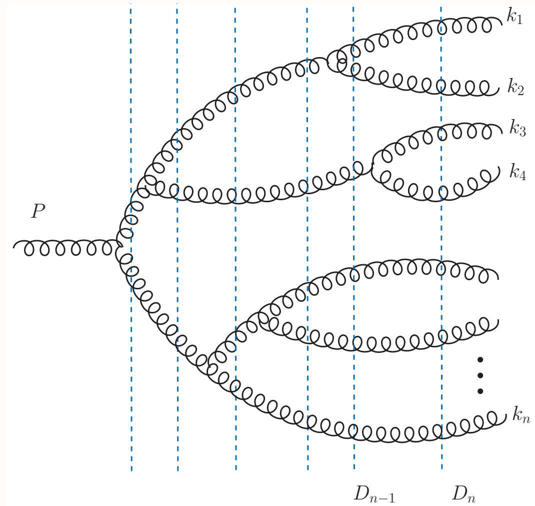
$$L^z = -(n-1)$$

Vanishes Because Maximum $|L^z| = n-2$

Light Front Analog of MHV rules

LF Proof of Parke-Taylor





$$m(1^-, 2^-, 3^+, \ldots, n^+) = ig^{n-2} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n-2\ n-1 \rangle \langle n-1\ n \rangle \langle n1 \rangle},$$

$$m(\pm, \pm, \ldots, \pm) = m(\mp, \pm, \pm, \ldots, \pm) = 0 .$$

Exact kinematics in the small x evolution of the color dipole and gluon cascade.
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e-Print: **arXiv:0901.4949** [hep-ph]

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

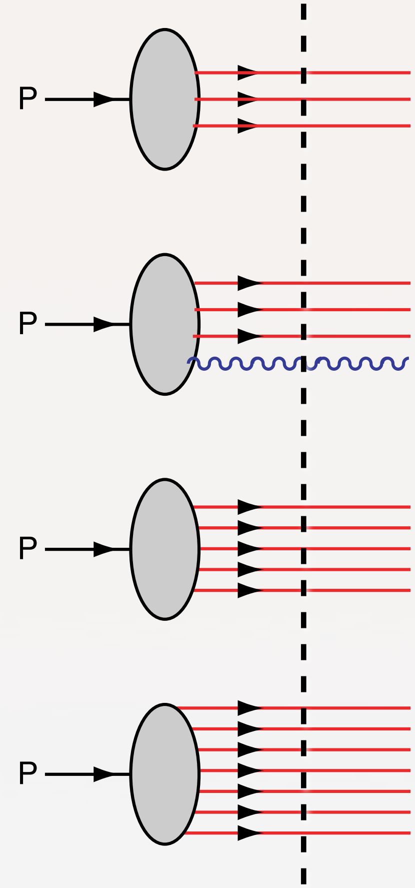
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

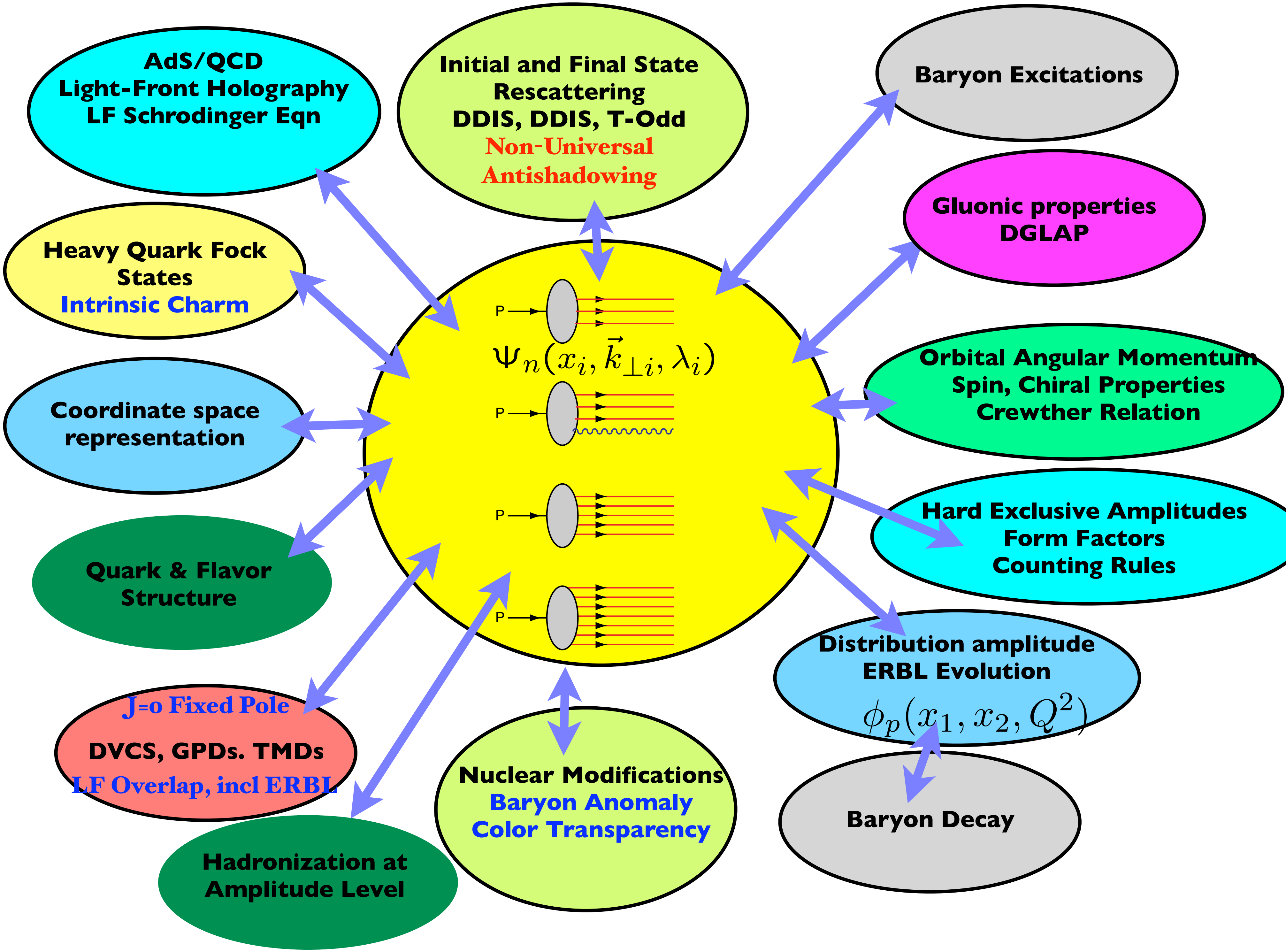
Intrinsic heavy quarks, $\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

Remarkable Features of Hadron Structure

- Valence quarks carry less than half of the proton's spin and momentum
- Non-zero quark orbital angular momentum
- Asymmetric sea: $\bar{u}(x) \neq \bar{d}(x)$ related to meson cloud
- Non-symmetric strange and antistrange sea $\Delta s(x) \neq \Delta \bar{s}(x)$ $\bar{s}(x) \neq s(x)$
- Intrinsic charm and bottom at high x
- Hidden-Color Fock states of the Deuteron

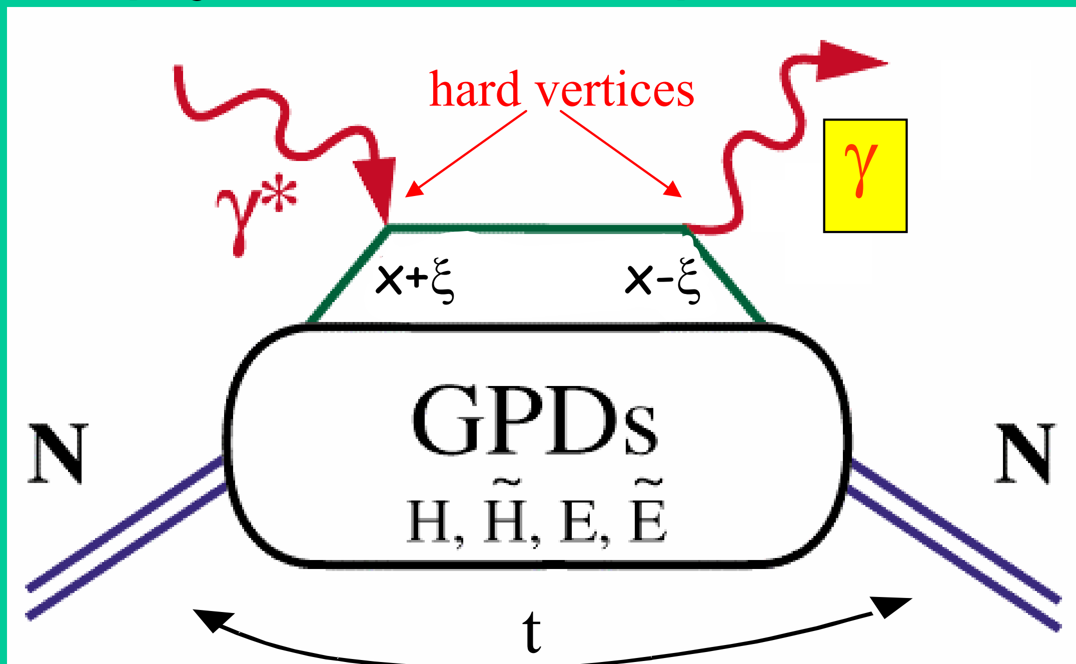
QCD and the LF Hadron Wavefunctions



GPDs & Deeply Virtual Exclusive Processes

"handbag" mechanism

Deeply Virtual Compton Scattering (DVCS)



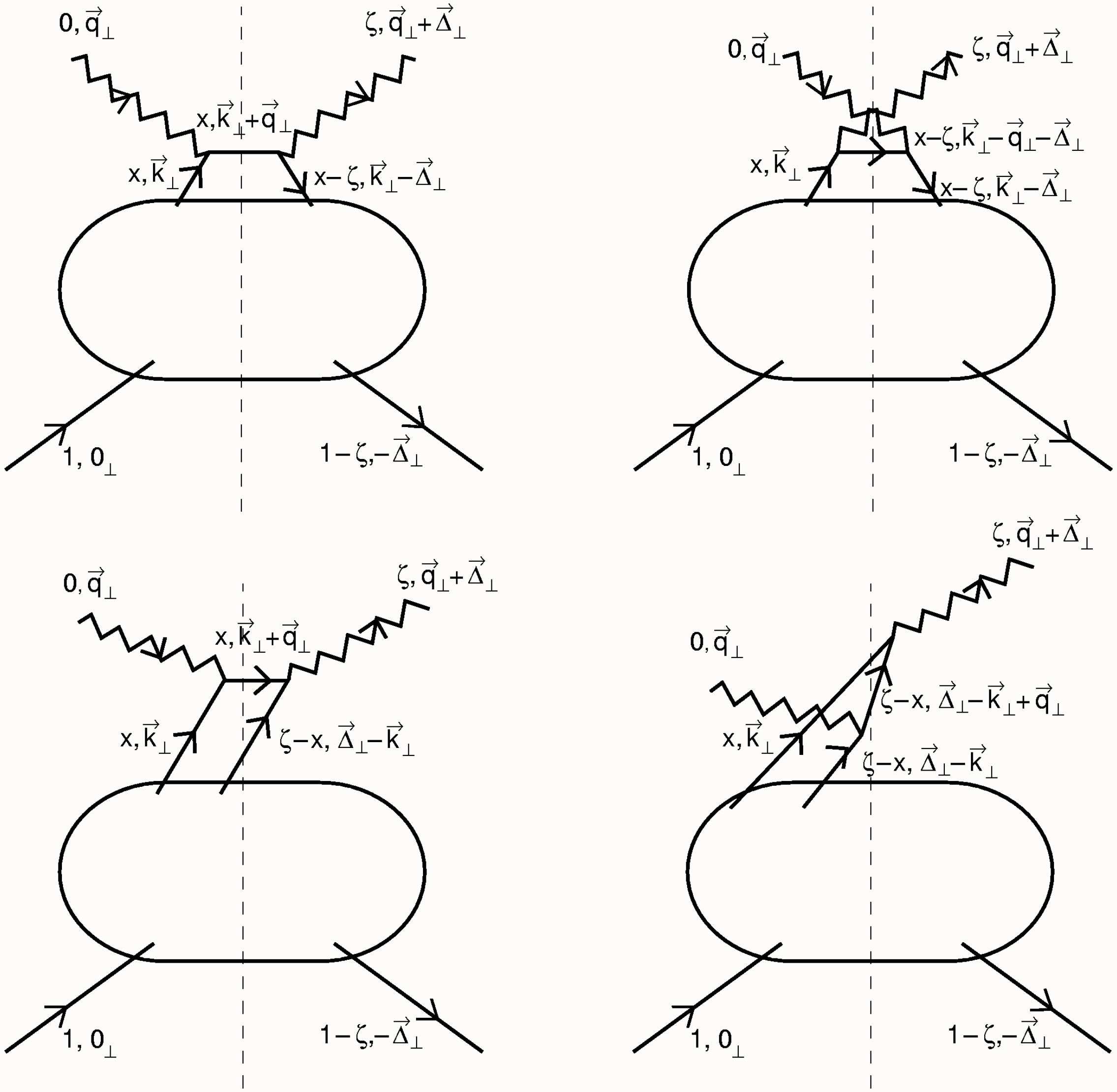
x – longitudinal quark momentum fraction

2ξ – longitudinal momentum transfer

$\sqrt{-t}$ – Fourier conjugate to transverse impact parameter

H(x,ξ,t), E(x,ξ,t), . .

$$\xi = \frac{x_B}{2-x_B}$$



Light-cone wavefunction representation of deeply virtual Compton scattering ☆

Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

Example of LFWF representation of GPDs (n => n)

Diehl,Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}}\frac{\Delta^1 - i\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x-x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

Link to DIS and Elastic Form Factors

DIS at $\xi=t=0$

$$H^q(x,0,0)=q(x),\quad -\bar{q}(-x)$$
$$\tilde{H}^q(x,0,0)=\Delta q(x),\quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int_{-1}^{1} dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \quad \text{Dirac f.f.}$$
$$\int_{-1}^{1} dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \quad \text{Pauli f.f.}$$
$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_{A,q}(t),\quad \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q,\, E^q,\, \tilde{H}^q,\, \tilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl,Hwang, sjb

Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx \left[H^q(x,\xi,0)+E^q(x,\xi,0)\right]$$

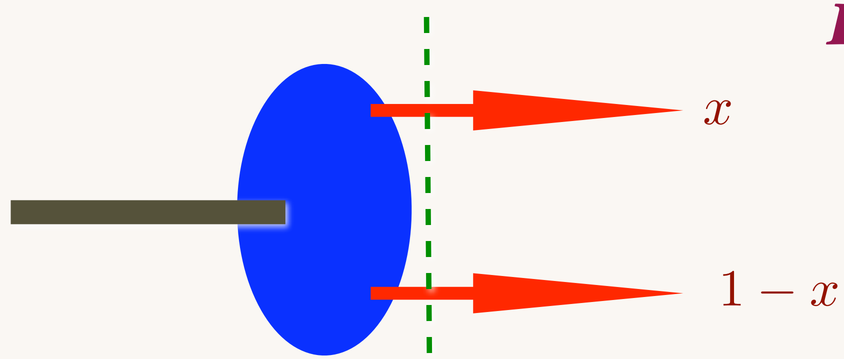
X. Ji, Phy.Rev.Lett.78,610(1997)

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



Fixed $\tau = t + z/c$

$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Perturbative QCD Analysis of Structure Functions at x ~ 1

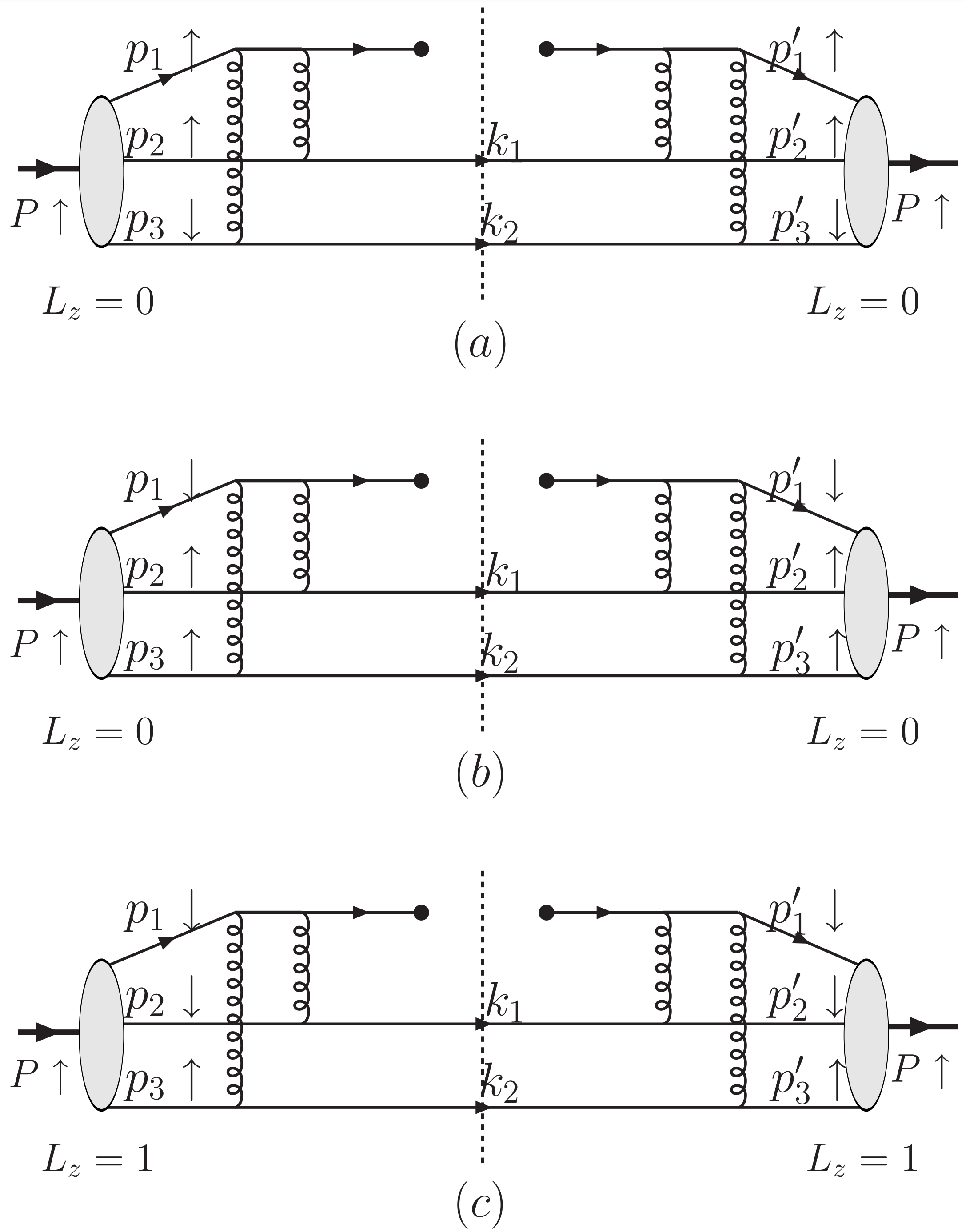
- Struck quark far off-shell at large x $$k_F^2 \simeq -\frac{k_\perp^2}{1-x}$$
- Lowest-order connected PQCD diagrams dominate
- Spectator counting rules $$(1-x)^{2n_s-1+2\Delta S_z}$$
- Helicity retention at large x
- Exclusive-Inclusive Connection

$$q^{+}(x) \propto (1-x)^3$$

$$q^{-}(x) \propto (1-x)^5 \log^2(1-x)$$

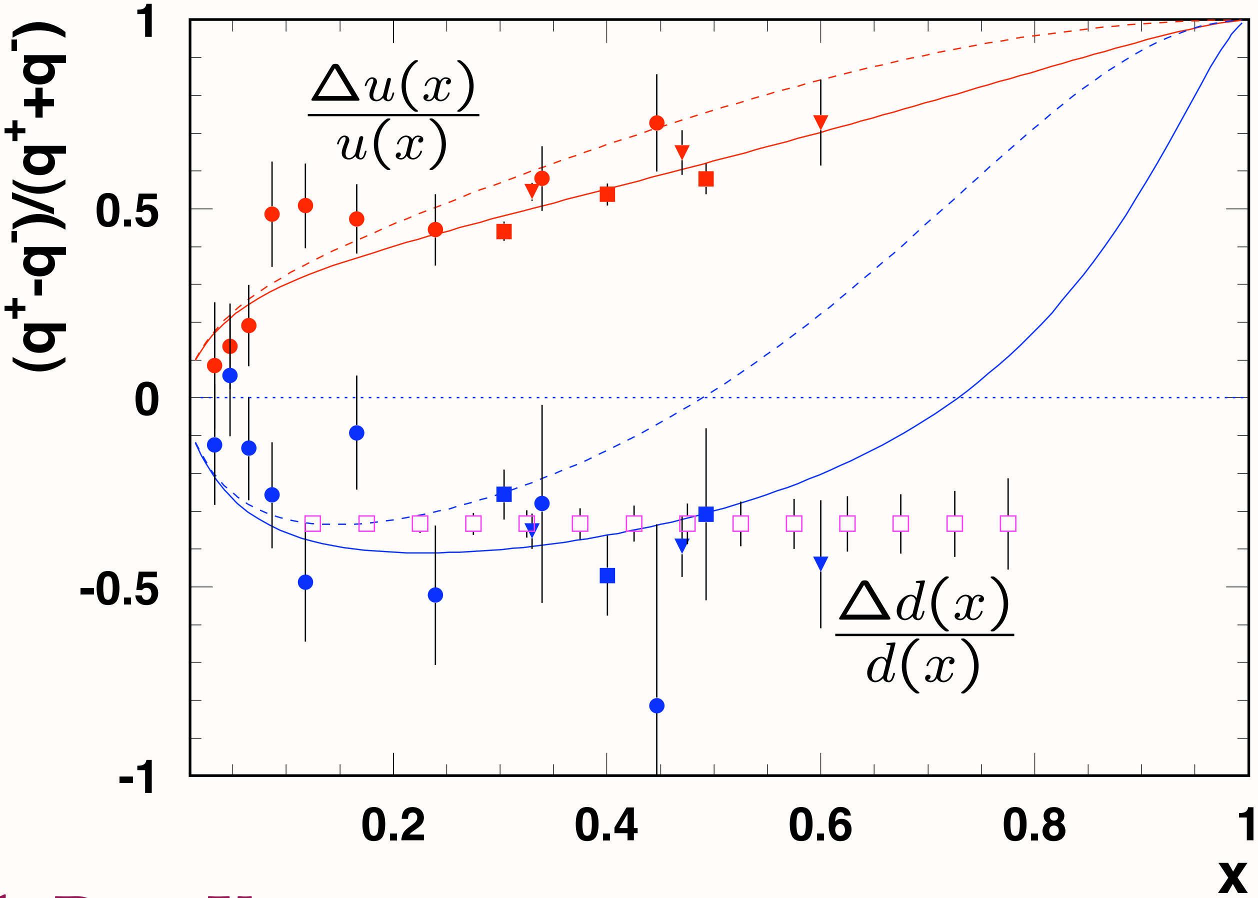
From nonzero orbital angular momentum

Avakian, sjb, Deur, Yuan



$$q^+(x) \propto (1-x)^3$$

$$q^-(x) \propto (1-x)^5 \log^2(1-x)$$

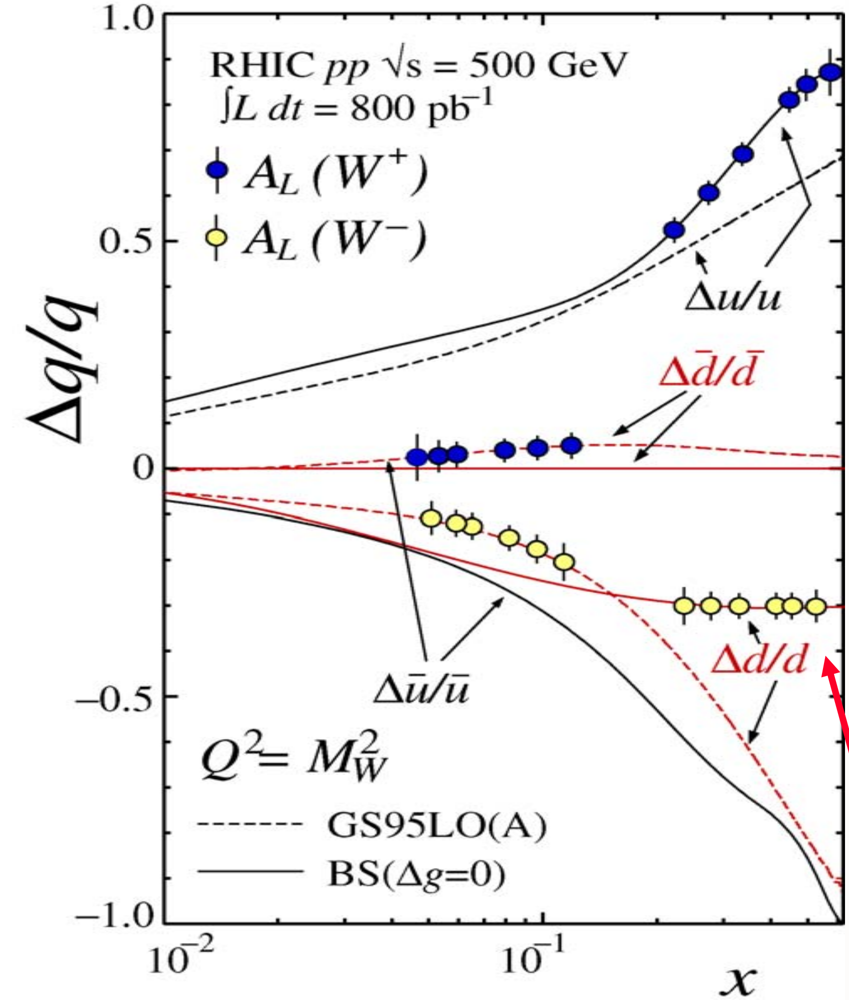


Avakian, sjb, Deur, Yuan

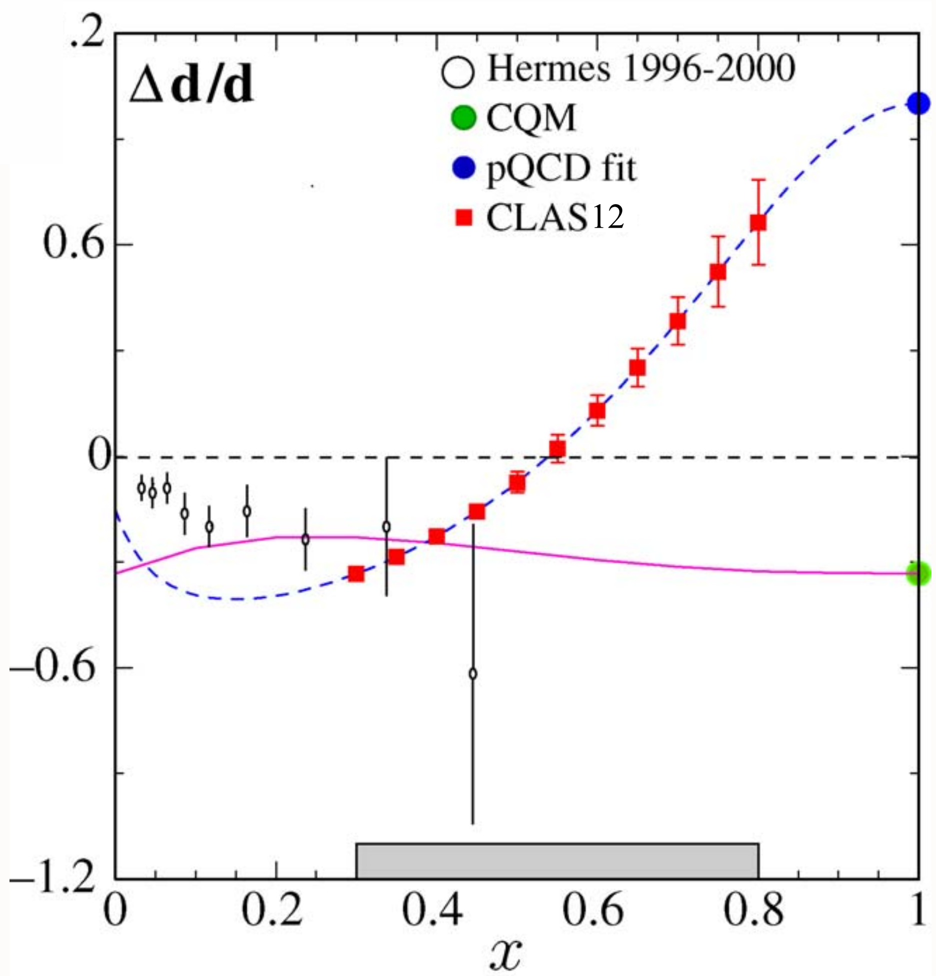
Similar to Ji, Balitsky, Yuan's PQCD analysis of $F_2(Q^2)/F_1(Q^2)$

At RHIC with W production

$$A_L^{W^+} \approx \frac{\Delta u(x_1)\,\bar{d}(x_2) - \Delta\bar{d}(x_1)\,u(x_2)}{u(x_1)\,\bar{d}(x_2) + \bar{d}(x_1)\,u(x_2)}$$



At JLab with 12 GeV upgrade



Stops below x=0.5 AND needs valence d(x)

Perturbative QCD Analysis of Structure Functions at x ~ 1

$$k_F^2 \propto \frac{-k_\perp^2}{1-x}$$

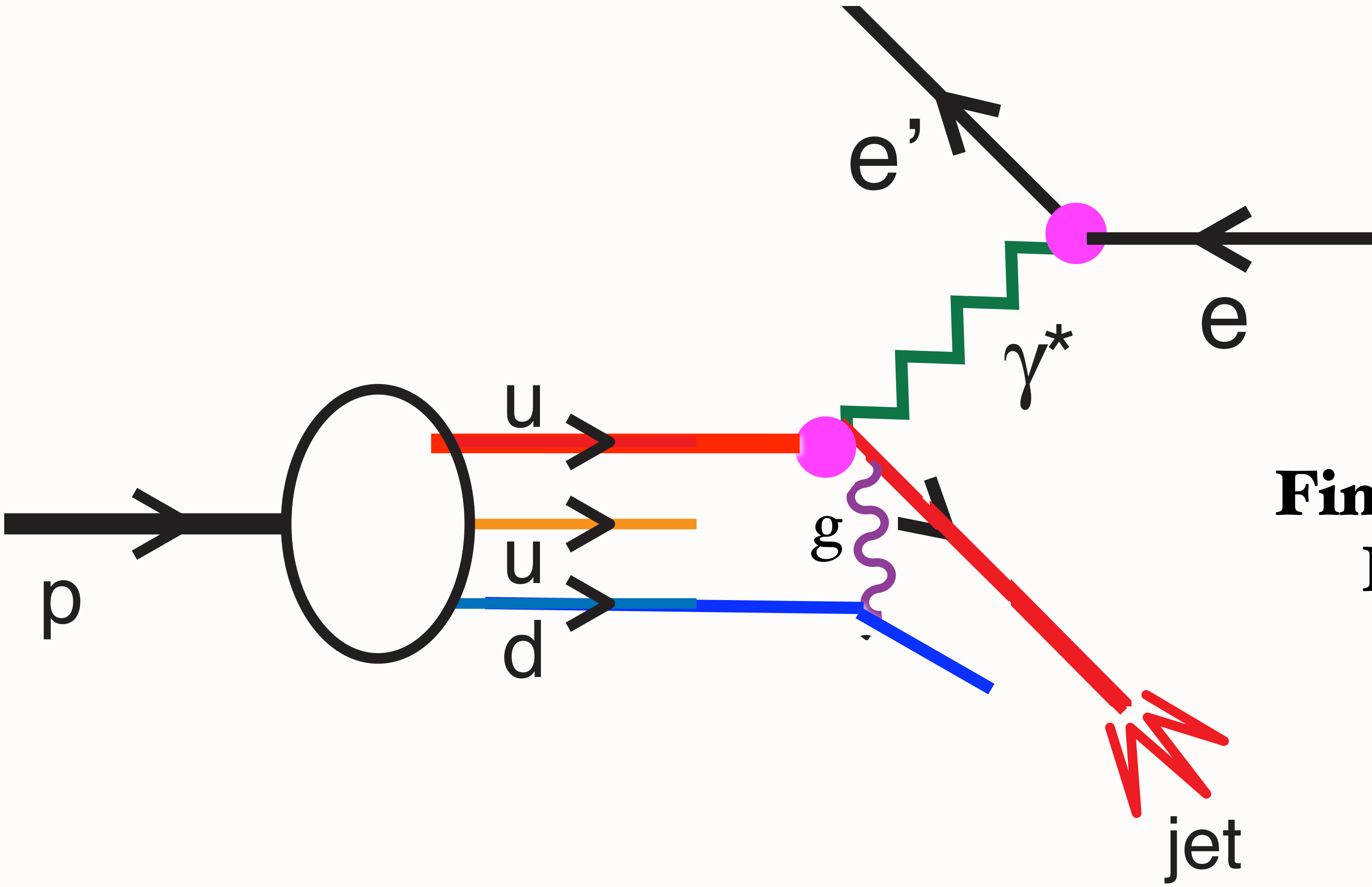
- Struck quark far off-shell at large x

- DGLAP evolution quenched due to off-shell struck quark

$$(1-x)^{P+\xi} \qquad \xi(Q^2, Q_0^2) = \frac{1}{4\pi}\int_{Q_0^2}^{Q^2} d\ell^2 \, \frac{\alpha_s(\ell^2)}{\ell^2 + \frac{k_\perp^2}{1-x}}$$

- Duality/ Exclusive-Inclusive connection at fixed W

Deep Inelastic Electron-Proton Scattering

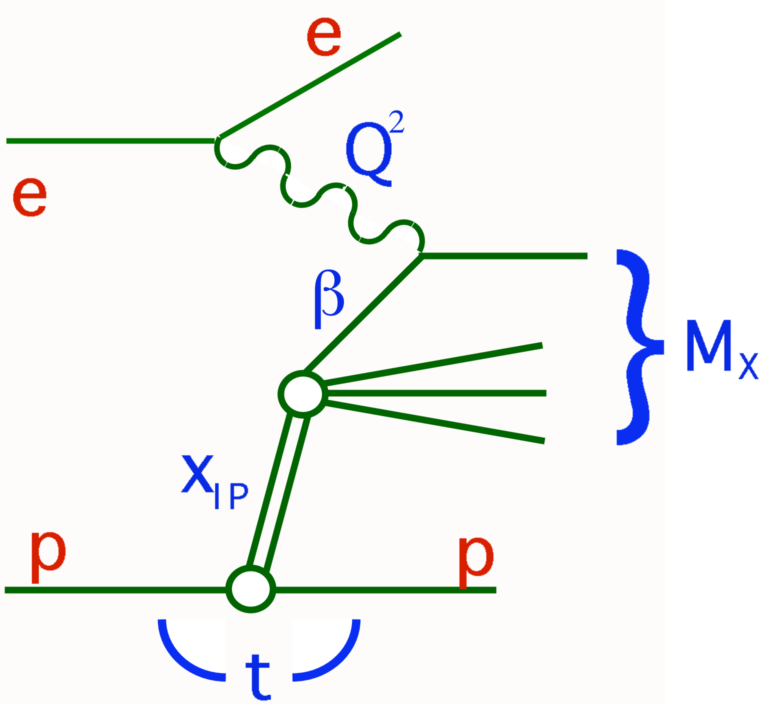


Final-State QCD Interaction

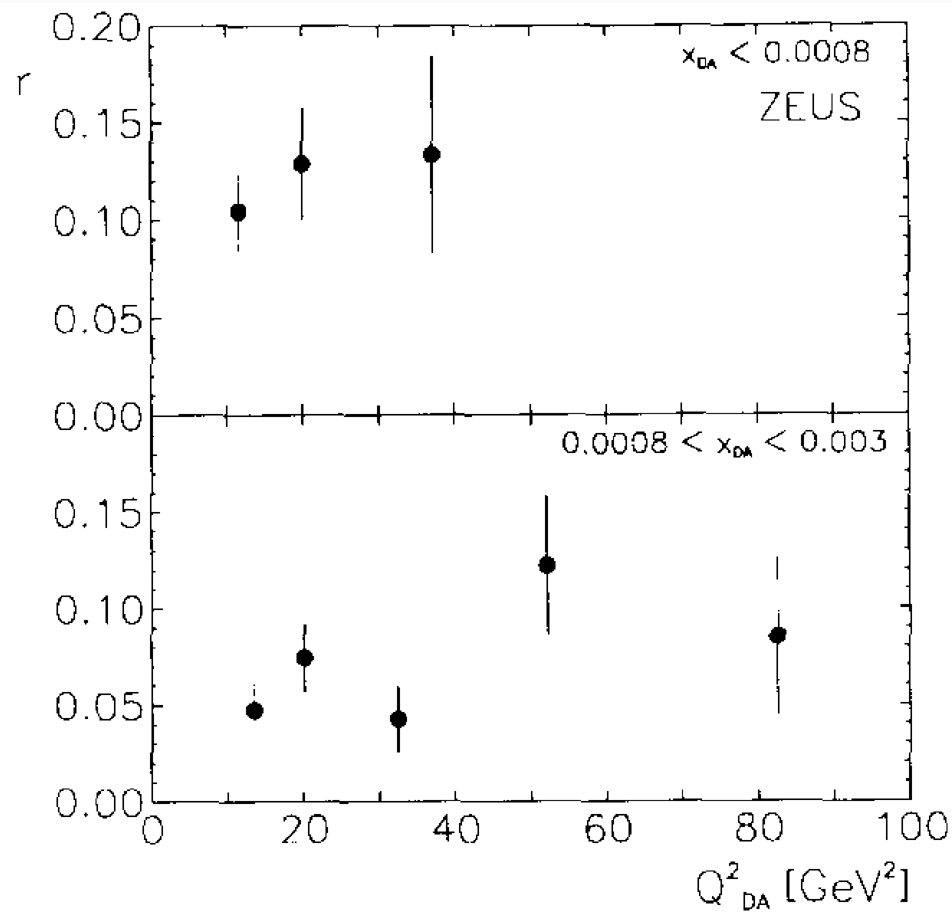
Conventional wisdom:

Final-state interactions of struck quark can be neglected

Remarkable observation at HERA



10% to 15% of DIS events are diffractive !

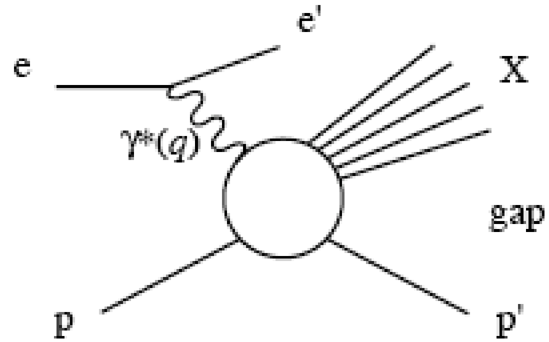


Fraction r of events with a large rapidity gap, $\eta_{max} < 1.5$, as a function of Q^2_{DA} for two ranges of x_{DA}. No acceptance corrections have been applied.

M. Derrick et al. [ZEUS Collaboration], Phys. Lett. B 315, 481 (1993).

DDIS

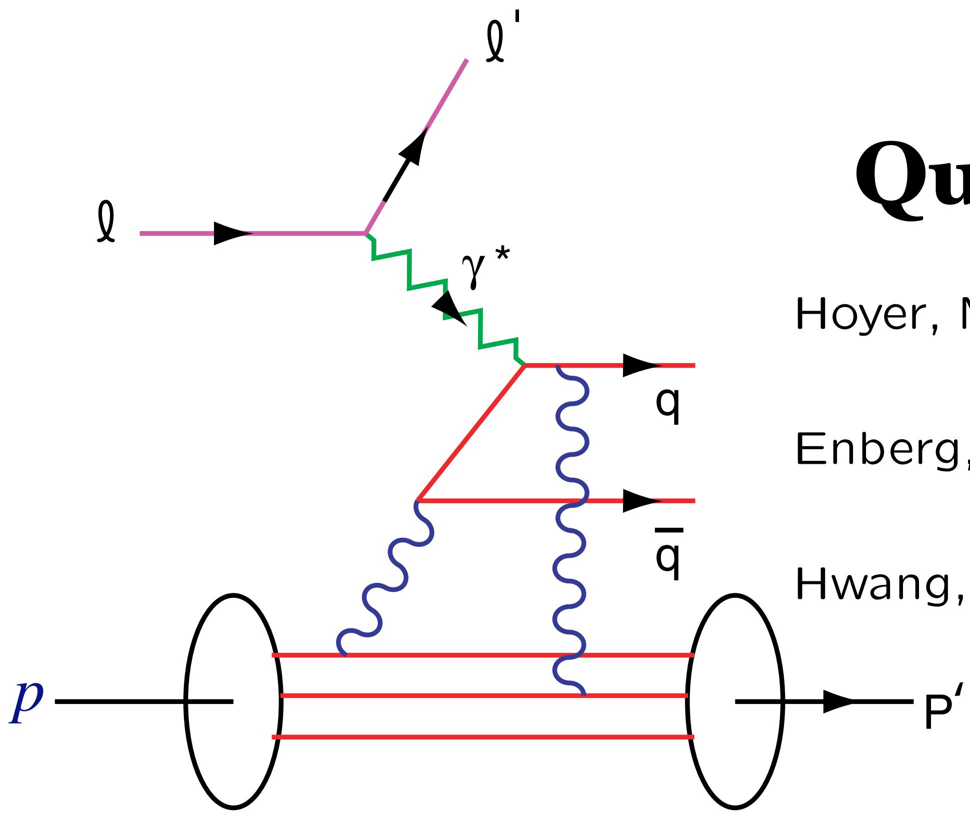
Diffractive Deep Inelastic Lepton-Proton Scattering



- In a large fraction ($\sim$ 10–15%) of DIS events, the proton escapes intact, keeping a large fraction of its initial momentum
- This leaves a large *rapidity gap* between the proton and the produced particles
- The t-channel exchange must be *color singlet* $\rightarrow$ a pomeron

Profound effect: target stays intact despite production of a massive system X

Final-State QCD Interaction Produces Diffractive DIS



Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHMPS)

Enberg, Hoyer, Ingelman, SJB

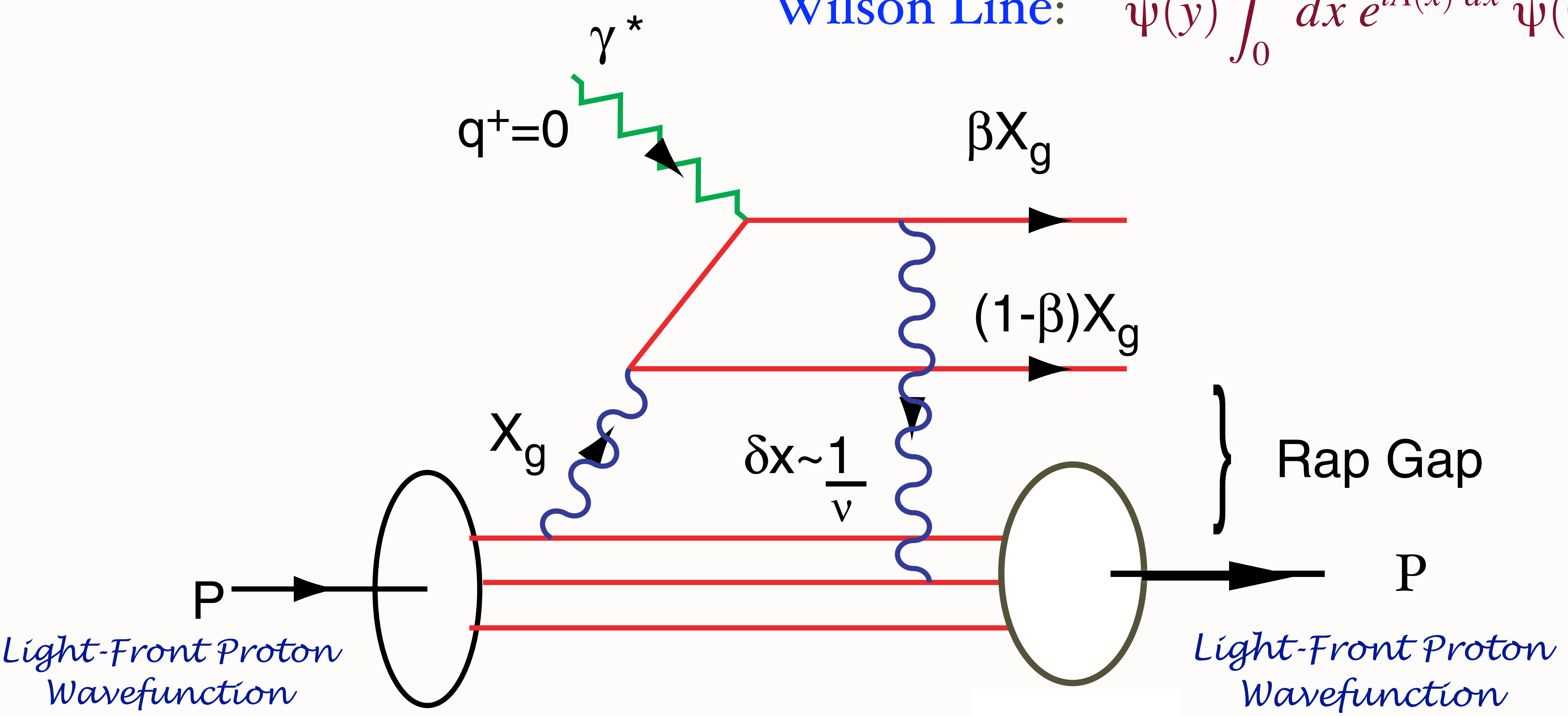
Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron

Hoyer, Marchal, Peigne, Sannino, sjb

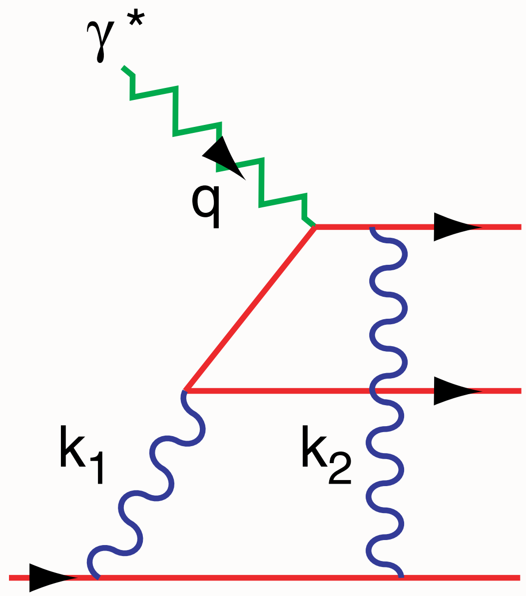
QCD Mechanism for Rapidity Gaps

Wilson Line: $\overline{\psi}(y) \int_0^y dx\, e^{iA(x)\cdot dx}\, \psi(0)$

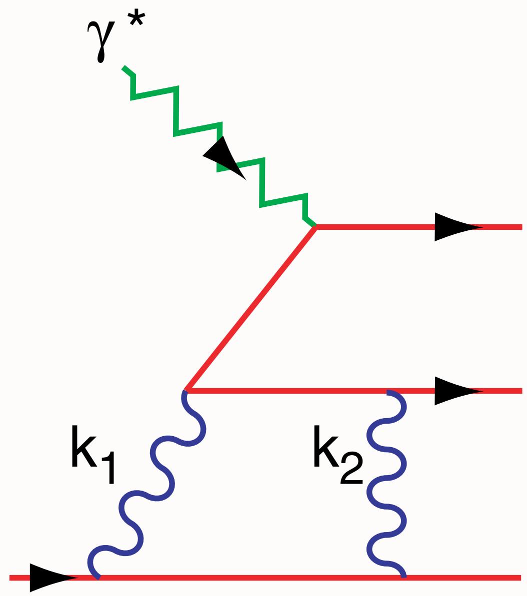


Reproduces lab-frame color dipole approach

Final State Interactions in QCD

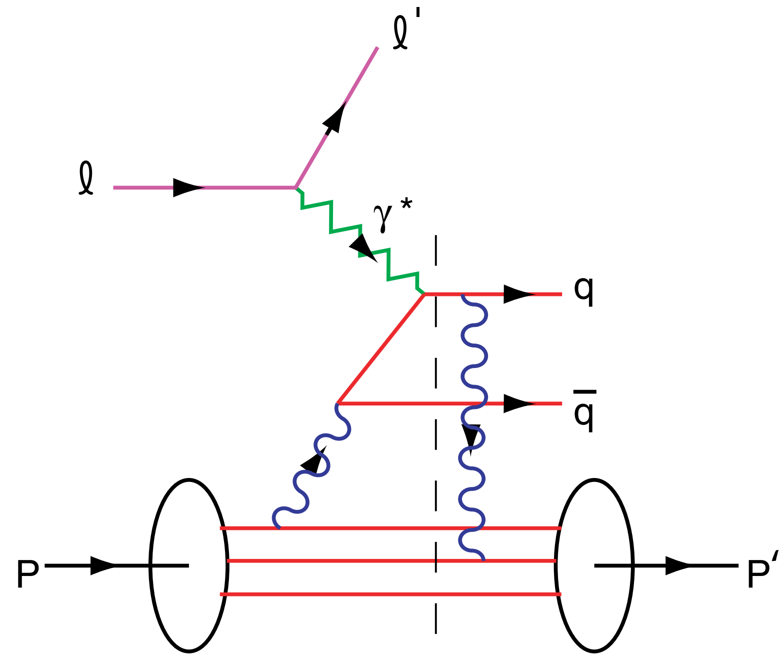


Feynman Gauge



Light-Cone Gauge

Result is Gauge Independent



Integration over on-shell domain produces phase i

Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target

Single-spin asymmetries

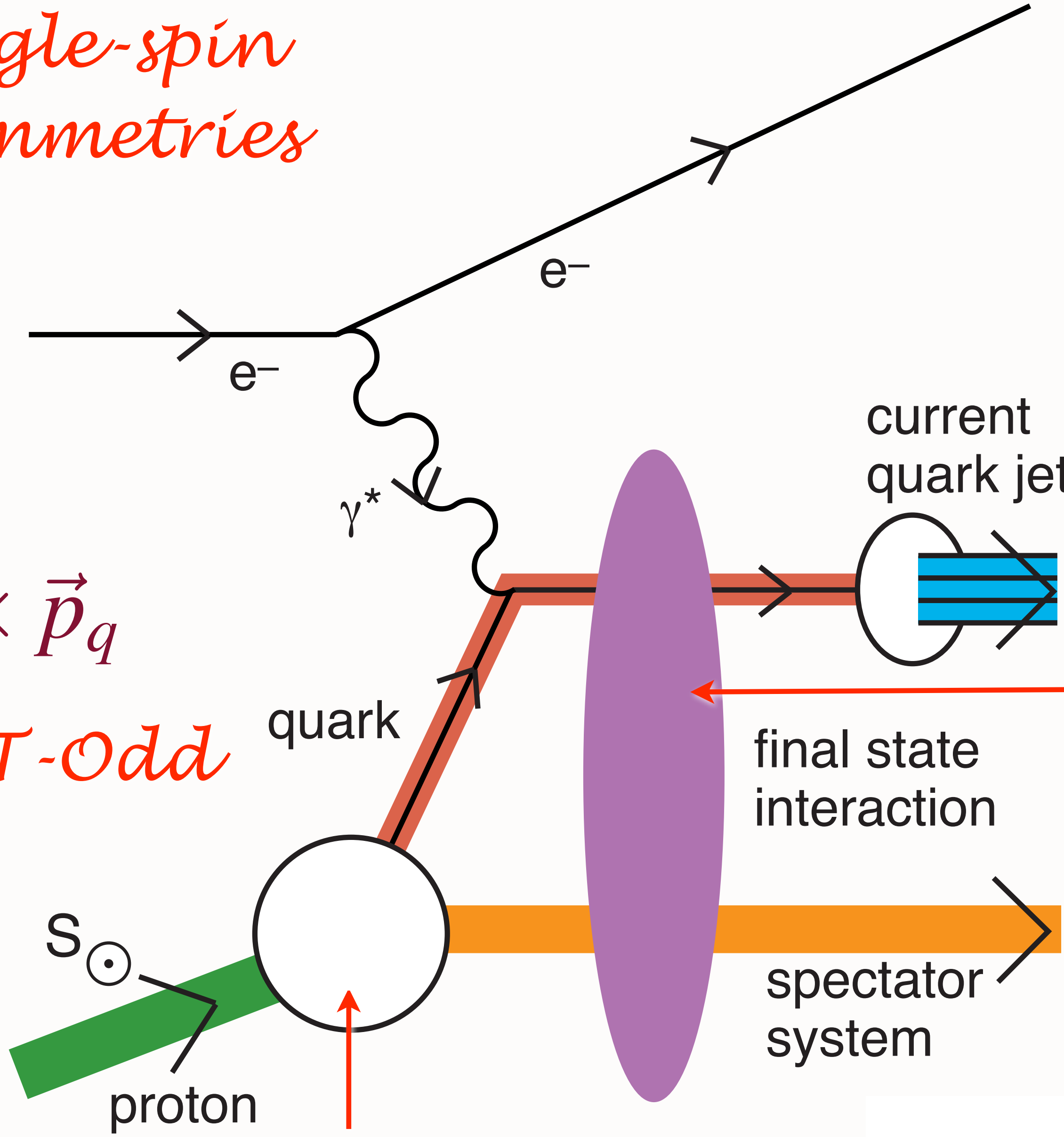
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

$$i\, \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd



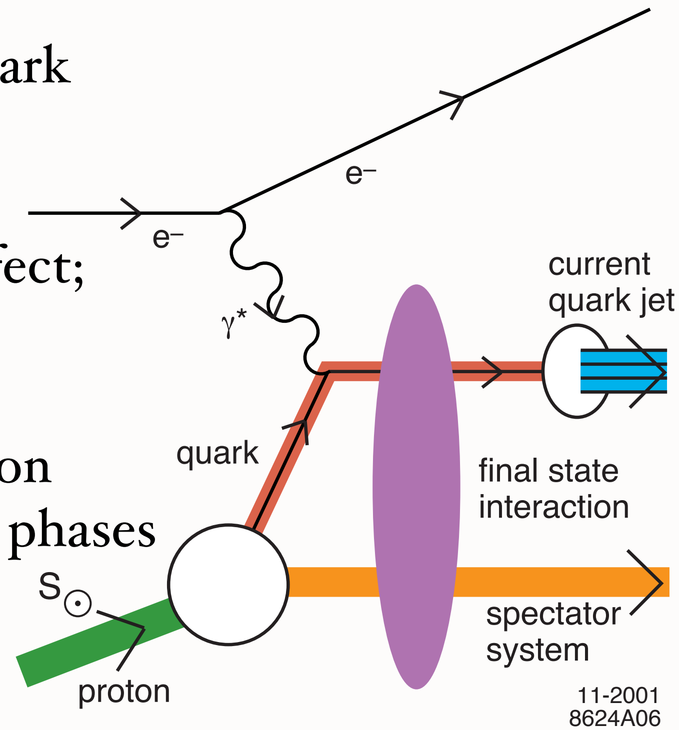
QCD S- and P- Coulomb Phases --Wilson Line

Light-Front Wavefunction S and P- Waves

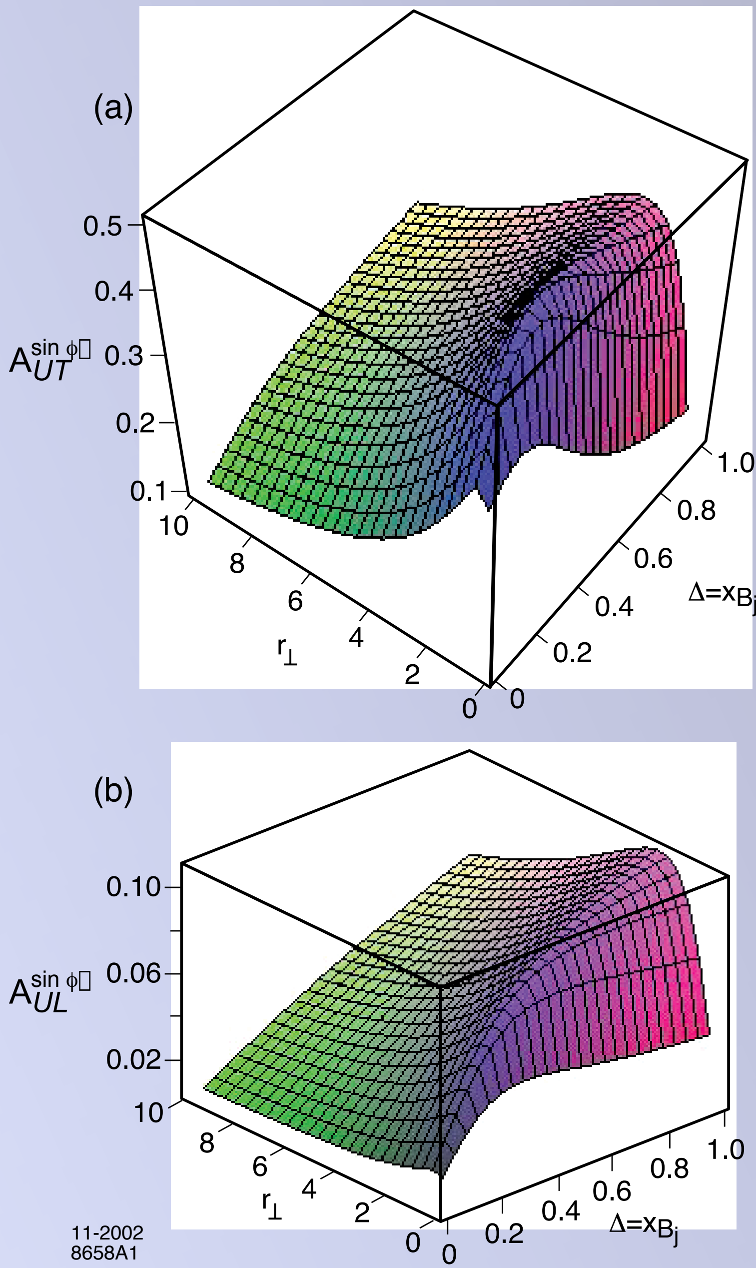
Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

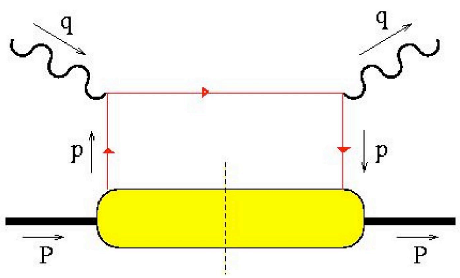
- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves; Wilson line effect; gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!



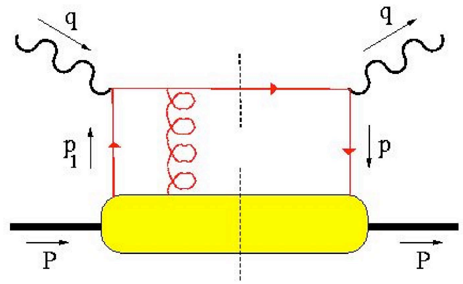
Prediction for Single-Spin Asymmetry



Hwang, Schmidt, sjb



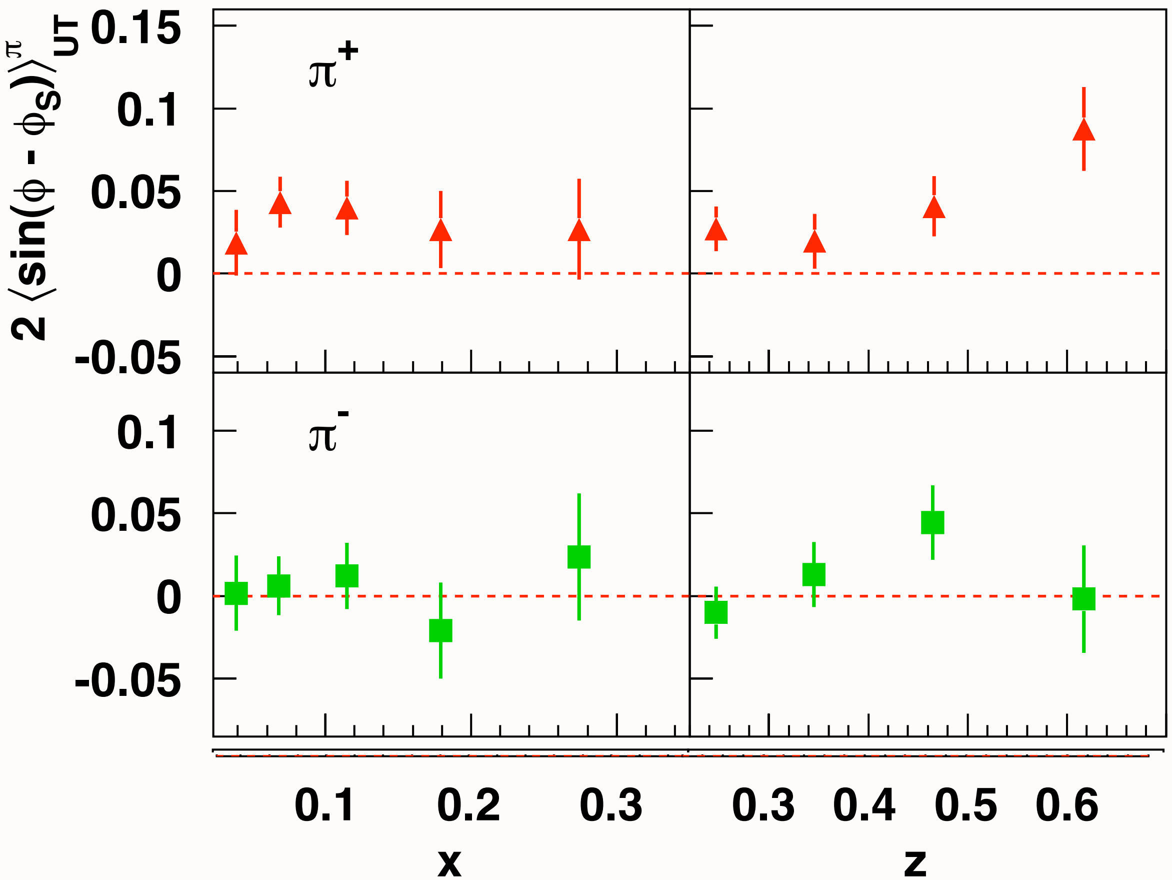
can interfere with



and produce a T-odd effect! (also need $L_z \neq 0$)

HERMES coll., A. Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002.

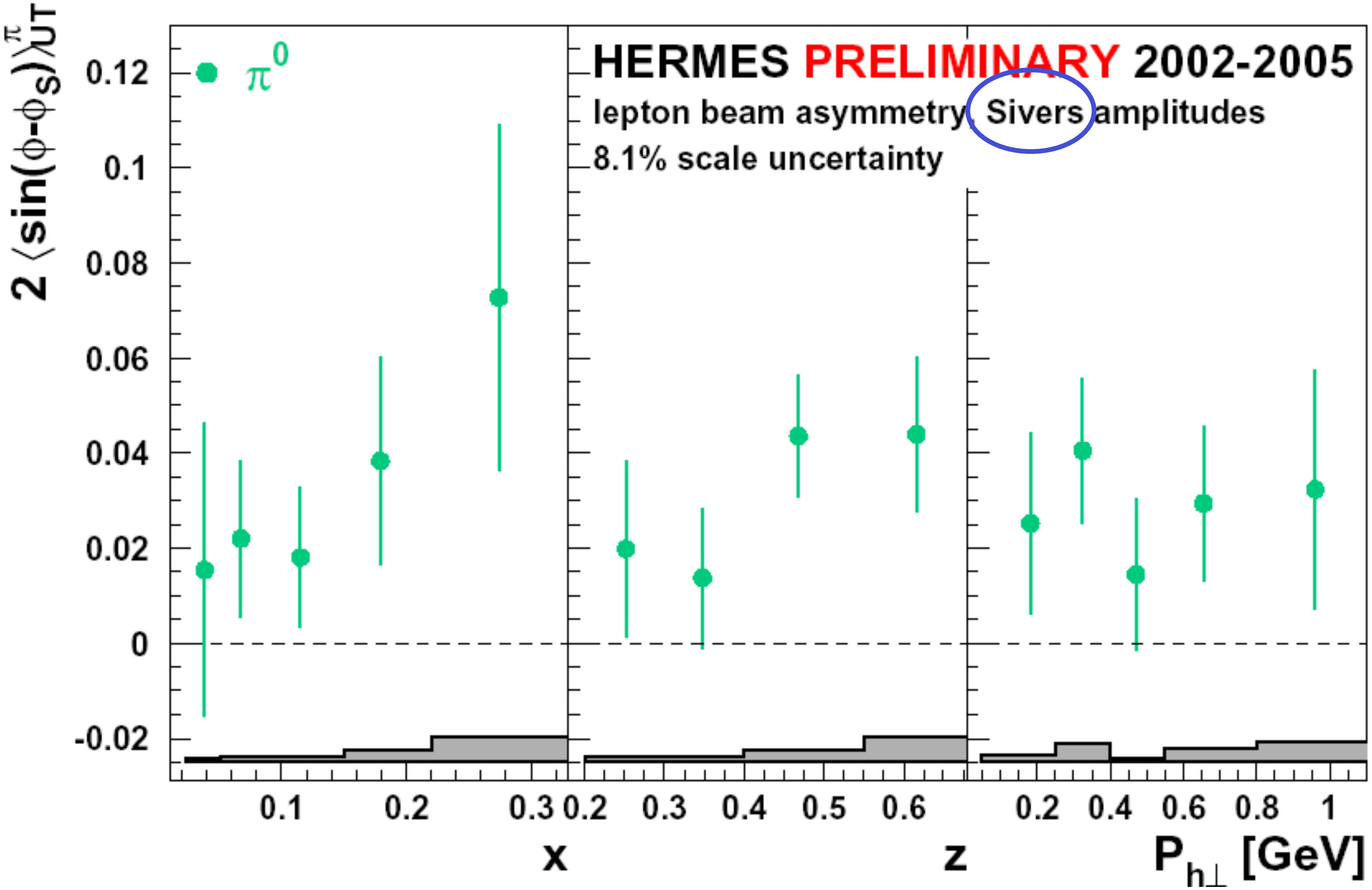
Sivers asymmetry from HERMES

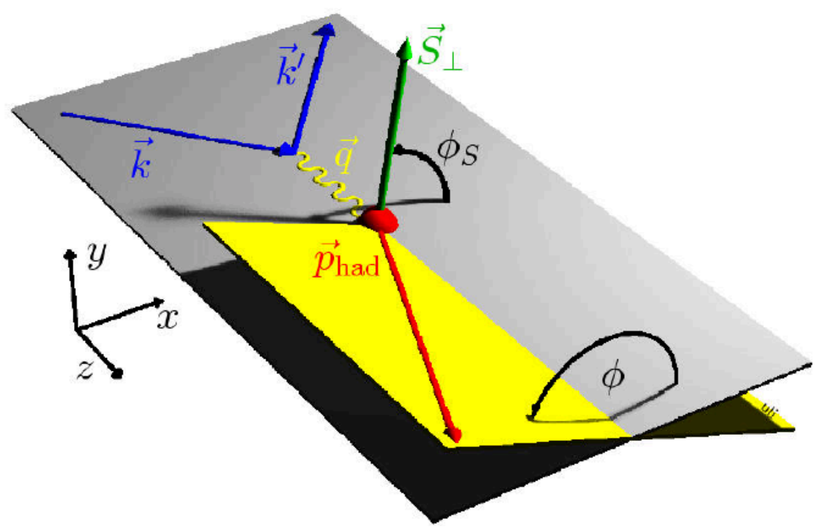


- First evidence for non-zero Sivers function!
- $\Rightarrow$ presence of non-zero **quark orbital angular momentum**!
- Positive for π^+... Consistent with zero for π^-...

Gamberg: Hermes data compatible with BHS model

Schmidt, Lu: Hermes charge pattern follow quark contributions to anomalous moment


HERMES PRELIMINARY 2002-2005
lepton beam asymmetry, Sivers amplitudes
8.1% scale uncertainty
π^0
$2\langle\sin(\phi-\phi_S)\rangle^{\pi}_{UT}$
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
0.1
0.2
0.3
0.2
0.3
0.4
0.5
0.6
0.2
0.4
0.6
0.8
1
x
z
$P_{h\perp}$ [GeV]

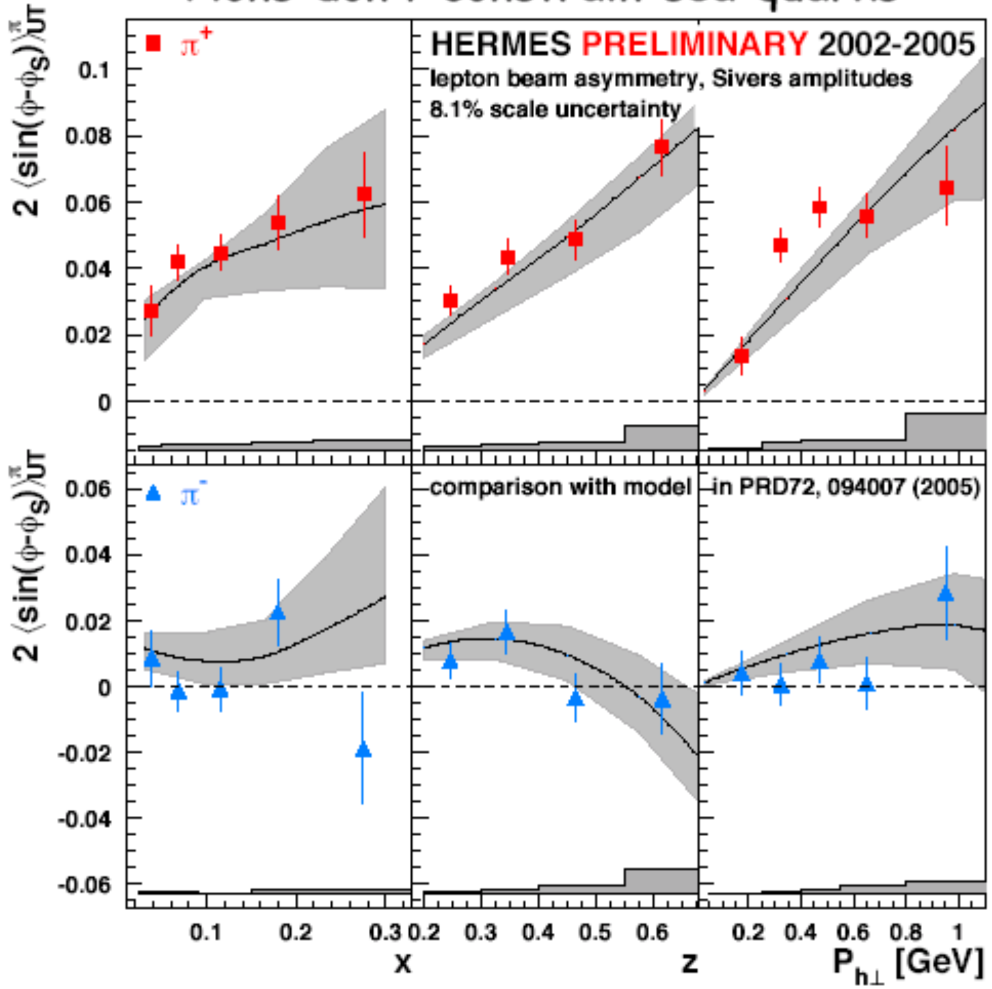

$\vec{k}'$
$\vec{S}_\perp$
$\vec{k}$
$\vec{q}$
ϕ_S
$\vec{p}_{\mathrm{had}}$
y
x
z
ϕ

M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin

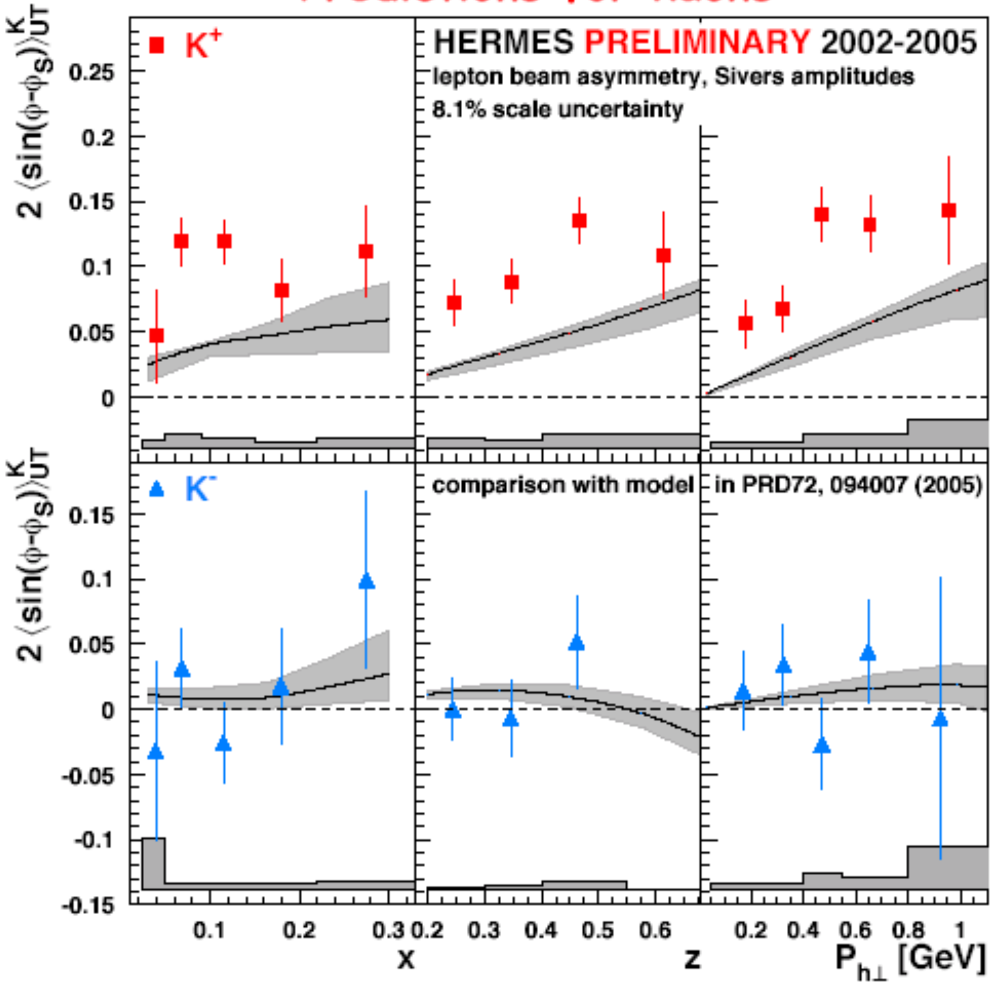
A fit of HERMES + COMPASS pion data,

information on u and d Sivers functions

Pions don't constrain sea quarks



Predictions for kaons



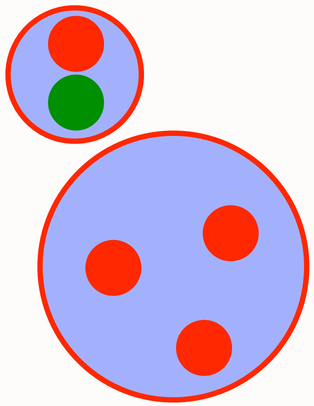
no sea contribution

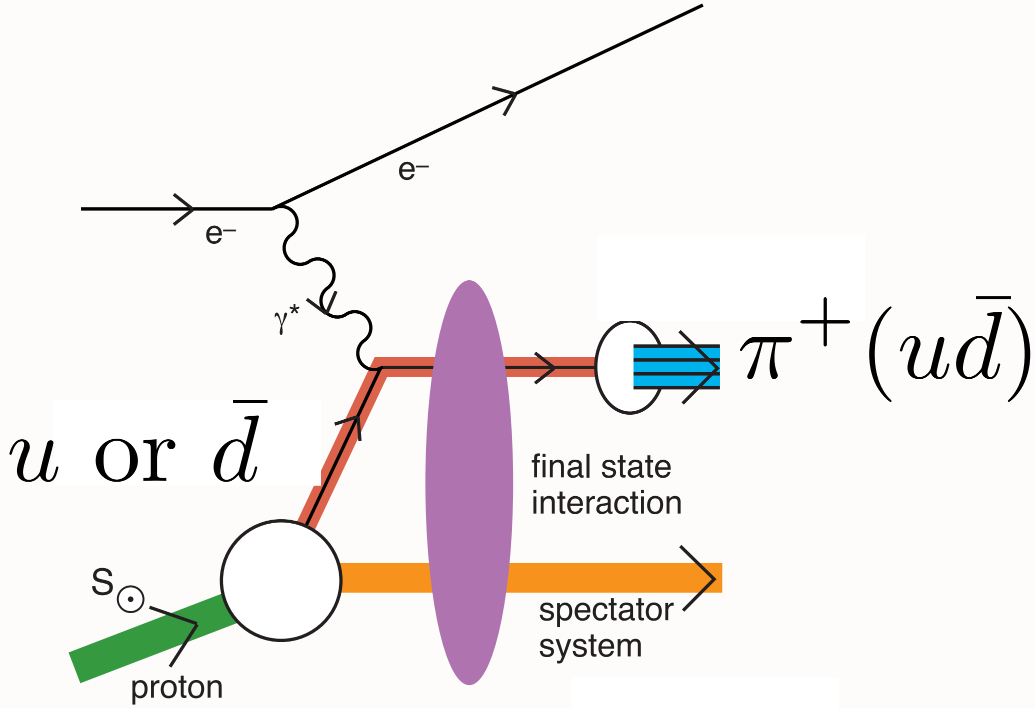
sea contribution?

(Kretzer fragmentation functions)

Gardner, sjb

Sea quarks carry orbital angular momentum





Sivers effect for $\pi^+(u\bar{d})$ reduced by $L_{\bar{d}}$ at low x

Sivers effect for $\pi^-(d\bar{u})$ reduced by $L_{\bar{u}}$ at low x

Sivers effect for $K^+(u\bar{s})$ increased by $L_{\bar{s}}$!

Estimate of $< L_q >$

Orbital functions	Song parameters	This paper
u quark	0.150	0.197 ± 0.02
d quark	0.025	-0.012 ± 0.01
s quark	0.025	0.015 ± 0.005
Sum of quarks	0.200	0.200 ± 0.02

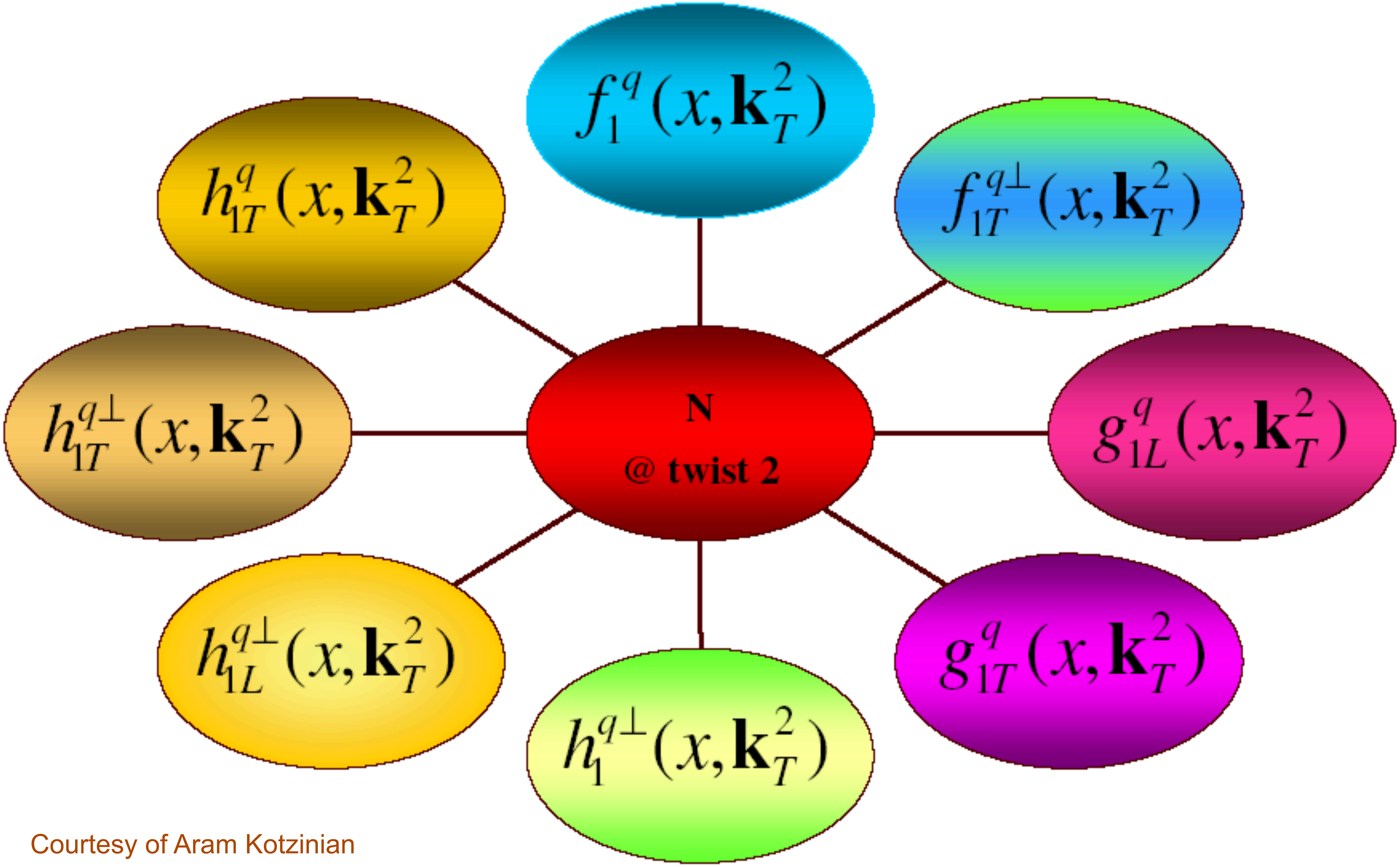
Orbital functions	Song parameters	This paper
$\bar{u}$ antiquark	0.017	0.015 ± 0.002
$\bar{d}$ antiquark	0.058	0.053 ± 0.006
$\bar{s}$ antiquark	0.025	0.022 ± 0.002
Sum of antiquarks	0.100	0.090 ± 0.01

Chiral Mechanisms Leading to Orbital Quantum Structures in the Nucleon.
Dennis Sivers (Portland Phys. Inst. & Michigan U.) . Apr 2007. 28pp.
e-Print: **arXiv:0704.1791** [hep-ph]

Physics of Rescattering

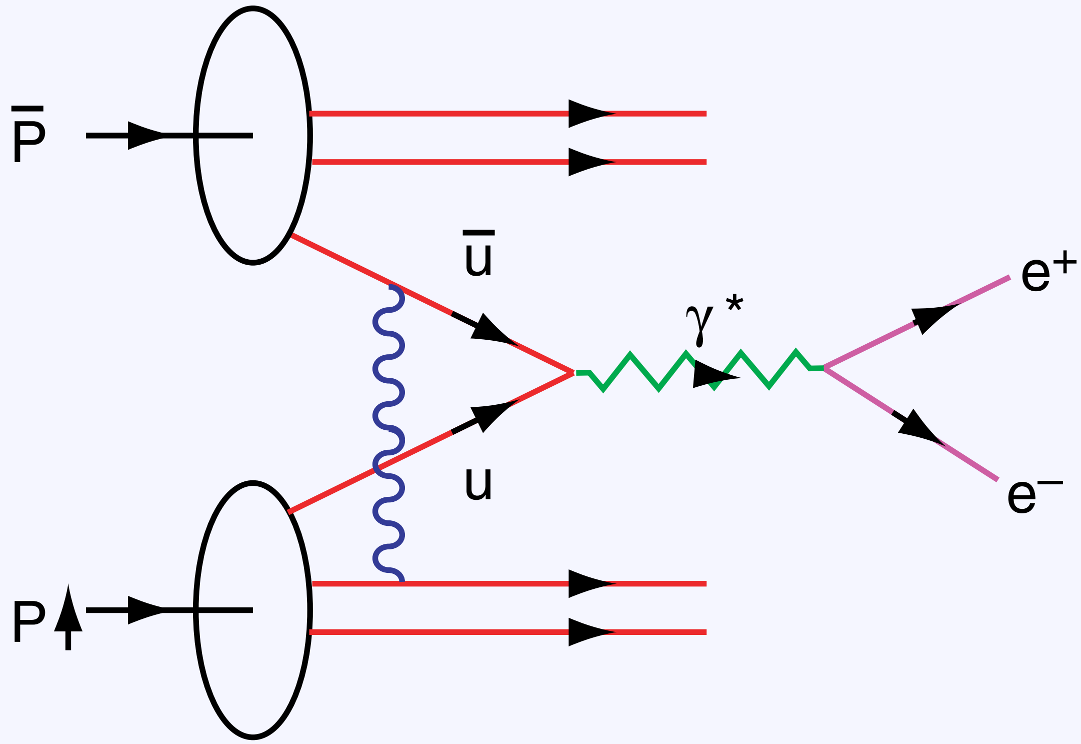
- Sivers Amplitude is Imaginary
- Phase comes from FSI
- Cannot be computed from wavefunction of proton in isolation!
- Phase requires QCD coupling in infrared
- Process dependent
- Input from hadron dynamics: Overlap of spin parallel and antiparallel LFWFS
- Same amplitudes which determine Pauli form factor

8 leading-twist spin-$k_\perp$ dependent distribution functions



Courtesy of Aram Kotzinian

Predict Opposite Sign SSA in DY !



Collins;
Hwang, Schmidt.
sjb

Single Spin Asymmetry In the Drell Yan Process

$\vec{S}_p \cdot \vec{\bar{p}} \times \vec{q}_{\gamma^*}$

Quarks Interact in the Initial State

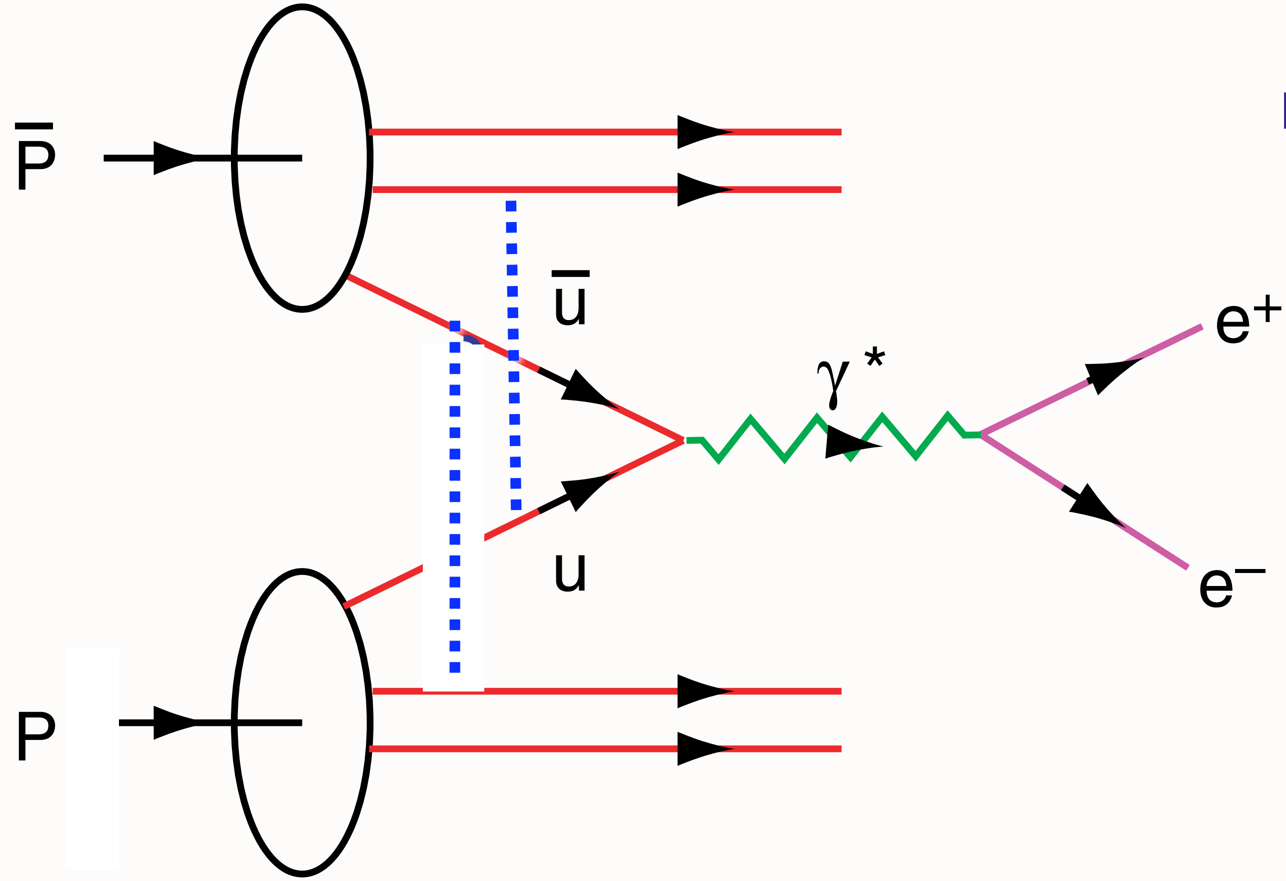
Interference of Coulomb Phases for *S* and *P* states

Produce Single Spin Asymmetry [Siver's Effect]Proportional to the Proton Anomalous Moment and α_s.

Opposite Sign to DIS! No Factorization

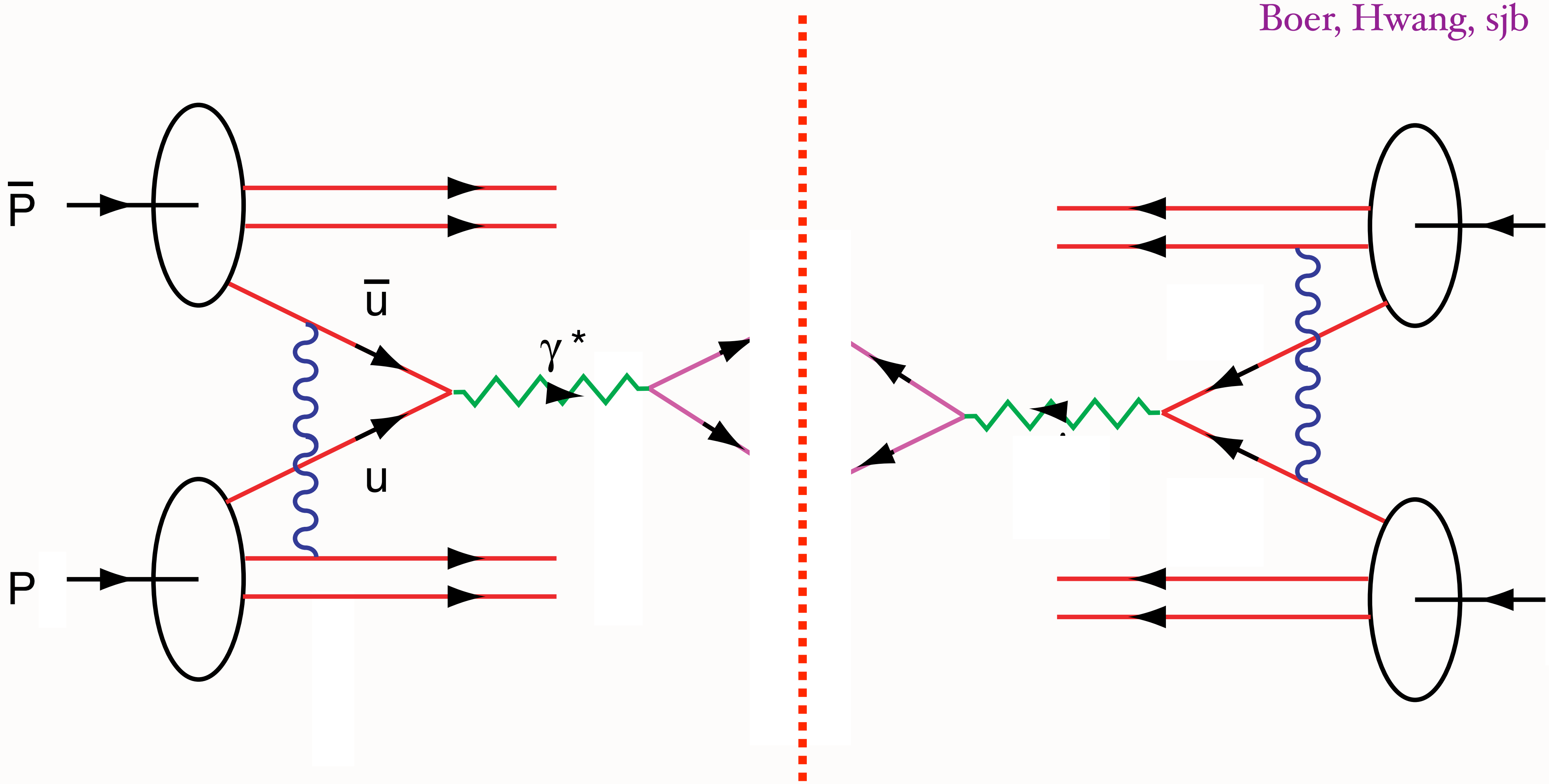
Boer, Hwang, sjb

F. Yuan et al



DY $\cos 2\phi$ correlation at leading twist from double ISI

Boer, Hwang, sjb

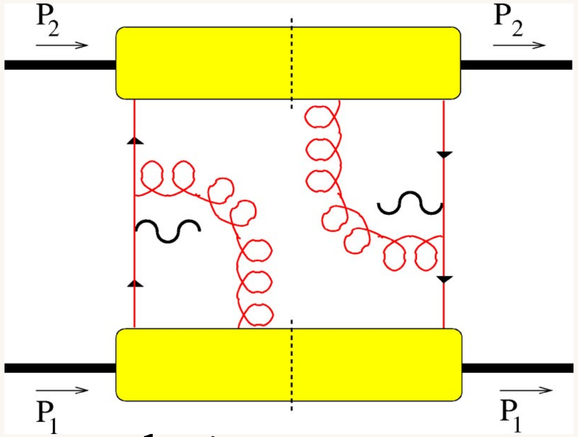


DY $\cos 2\phi$ correlation at leading twist from double ISI

Anomalous effect from Double ISI in Massive Lepton Production

Boer, Hwang, sjb

$\cos 2\phi$ correlation



- Leading Twist, valence quark dominated
- Violates Lam-Tung Relation!
- Not obtained from standard PQCD subprocess analysis
- Normalized to the square of the single spin asymmetry in semi-inclusive DIS
- No polarization required
- Challenge to standard picture of PQCD Factorization

Double Initial-State Interactions generate anomalous $\cos 2\phi$

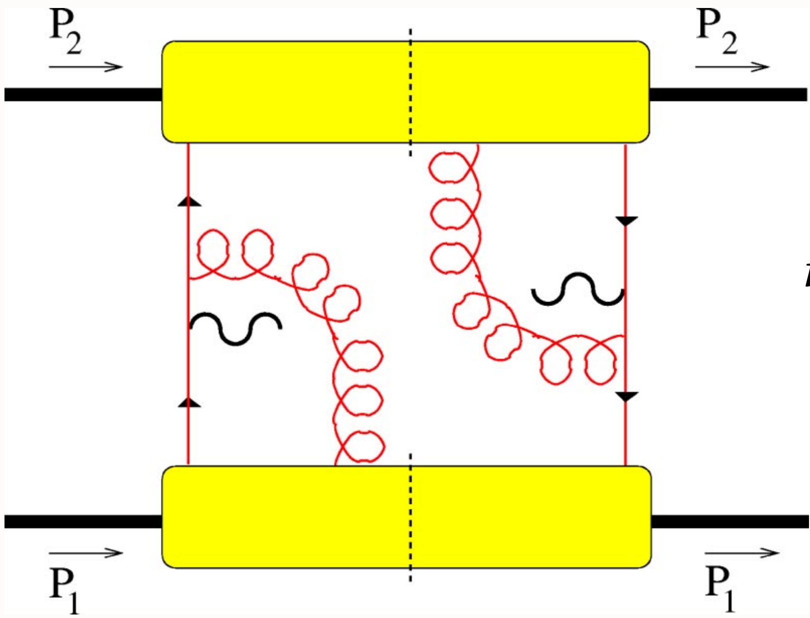
Boer, Hwang, sjb

Drell-Yan planar correlations

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \, \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right)$$

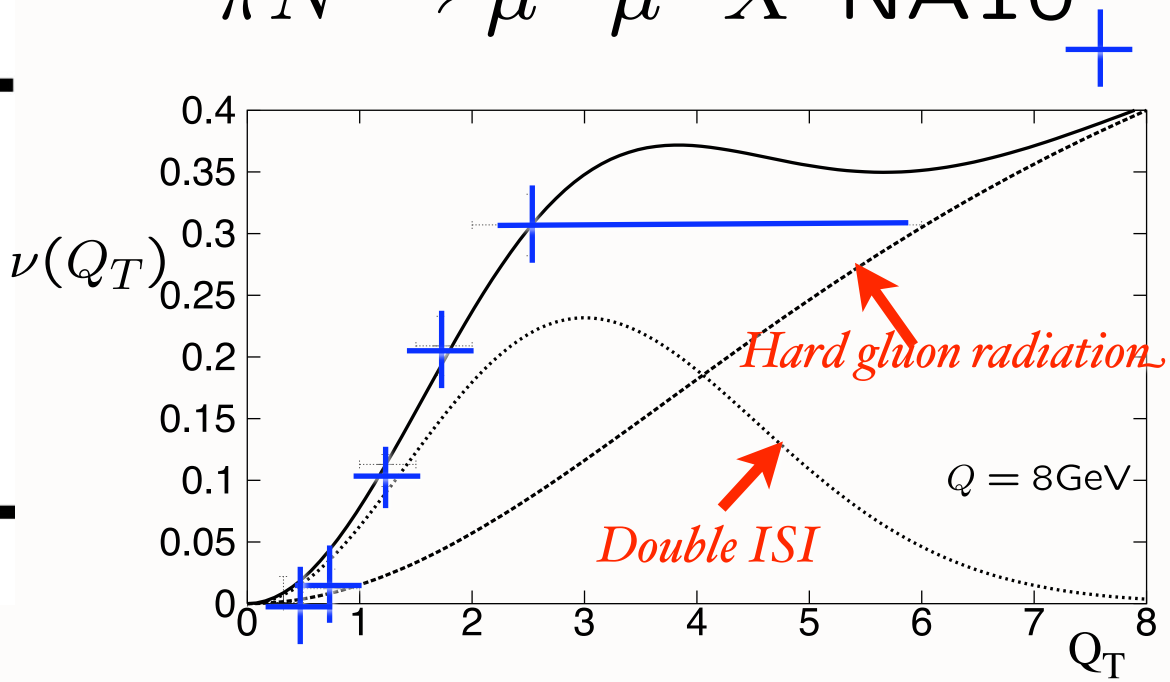
PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^{\perp}(\pi)\, h_1^{\perp}(N)$$

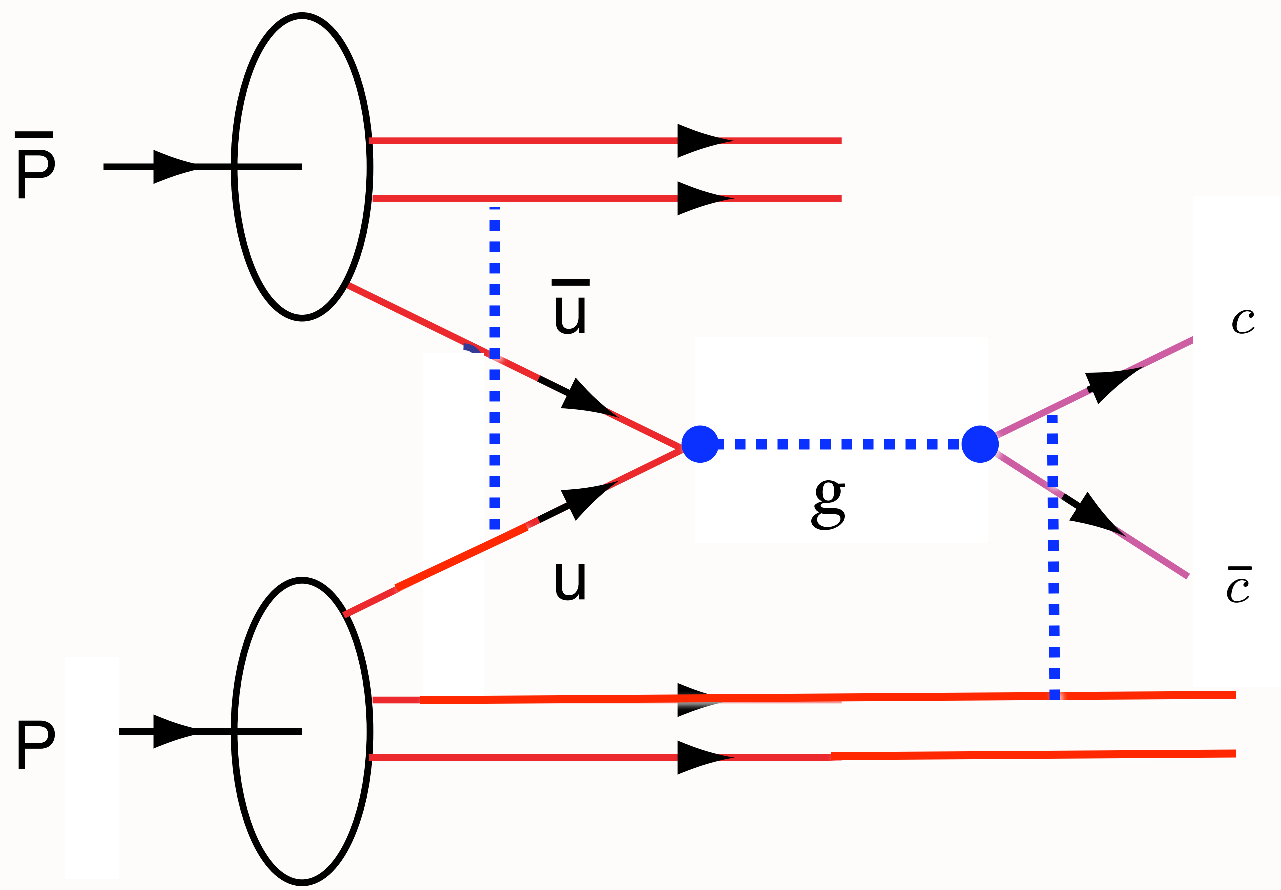


Violates Lam-Tung relation!

$\pi N \to \mu^+\mu^- X$ NA10



Model: Boer,

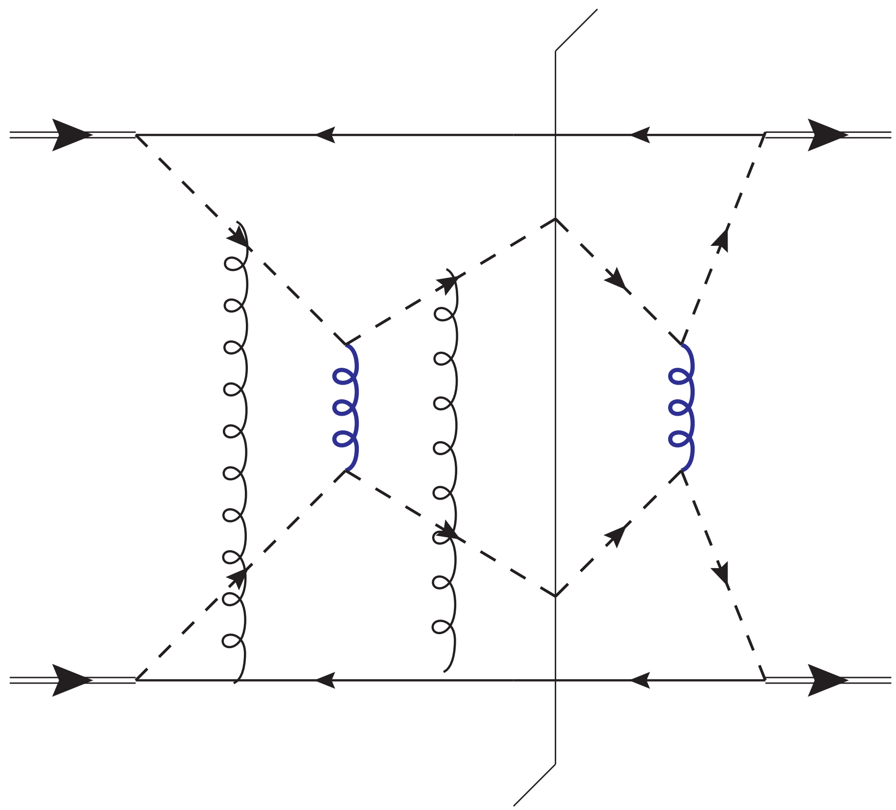


Problem for factorization when both ISI and FSI occur

Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

John Collins, Jian-Wei Qiu . ANL-HEP-PR-07-25, May 2007.

e-Print: **arXiv:0705.2141** [hep-ph]

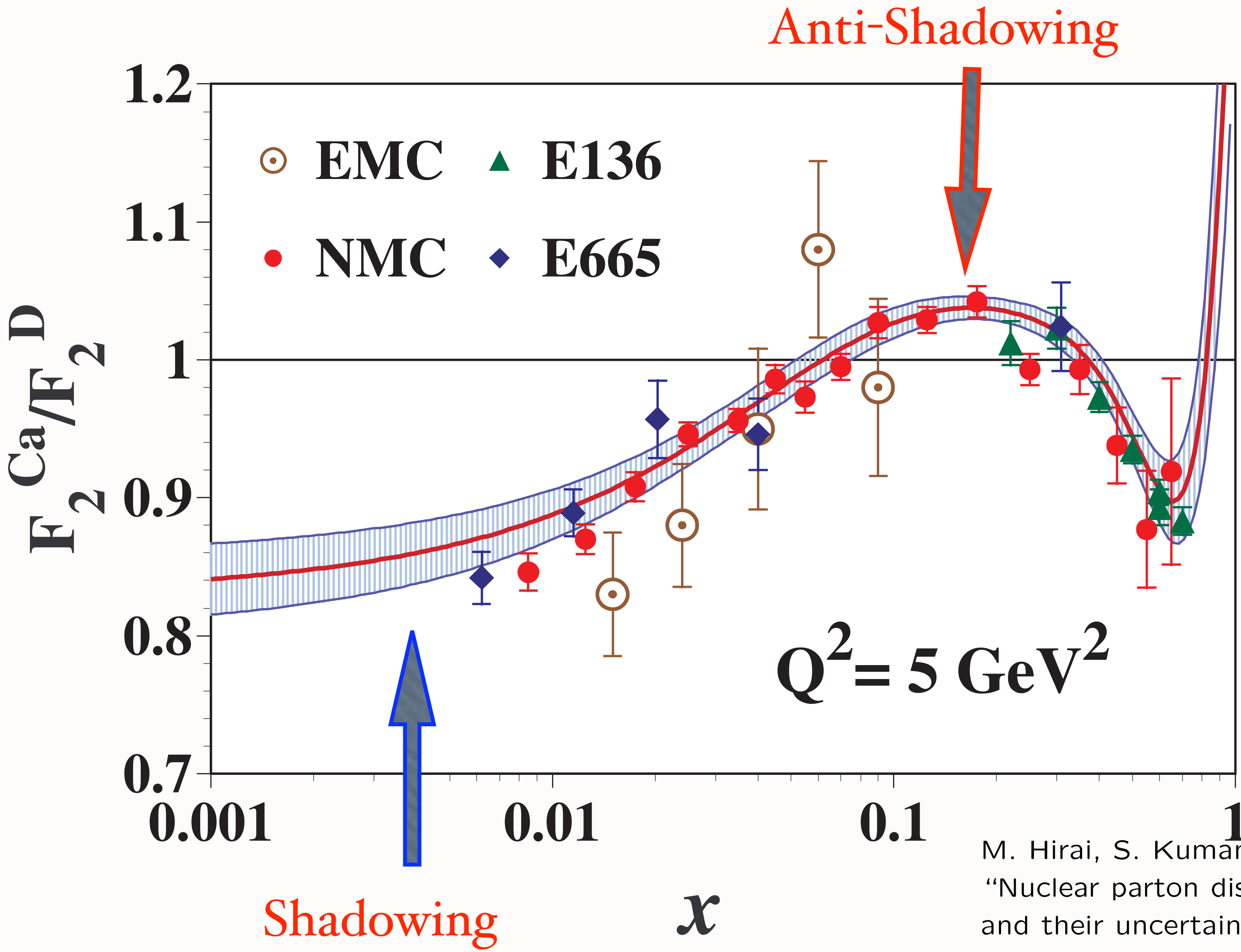


The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.

Physics of Rescattering

- Sivers Asymmetry and Diffractive DIS: New Insights into Final State Interactions in QCD
- Origin of Hard Pomeron
- Structure Functions not Probability Distributions!

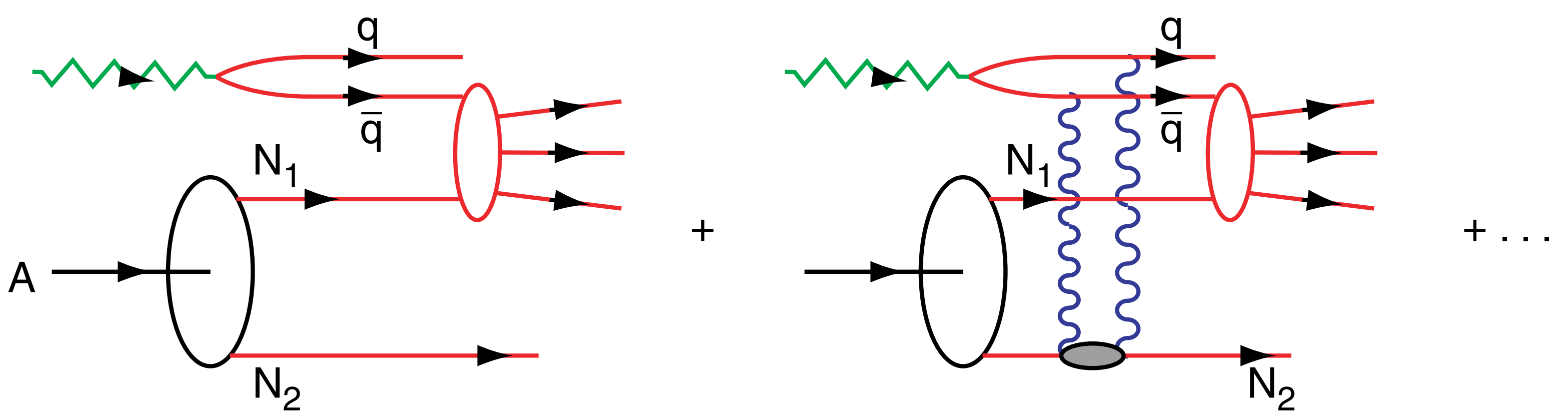
  Not square of LFWFs
- T-odd SSAs, Shadowing, Antishadowing
- Diffractive dijets/ trijets, doubly diffractive Higgs
- Novel Effects: Color Transparency, Color Opaqueness, Intrinsic Charm, Odderon



M. Hirai, S. Kumano and T. H. Nagai, "Nuclear parton distribution functions and their uncertainties," Phys. Rev. C **70**, 044905 (2004) [arXiv:hep-ph/0404093].

Stodolsky
Pumplin, sjb
Gribov

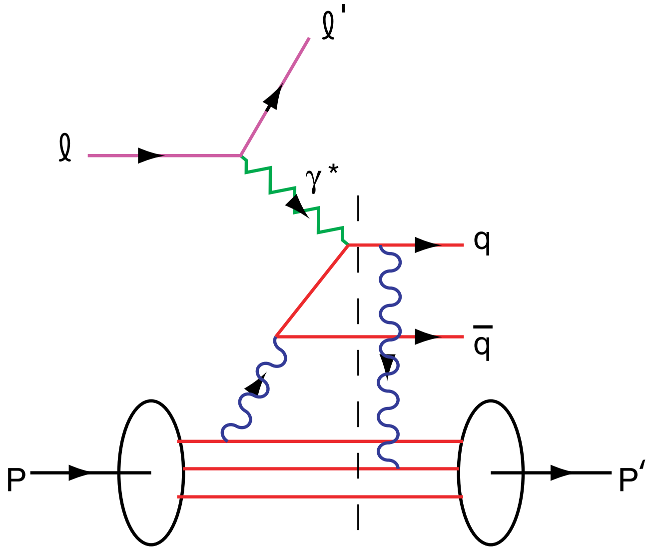
Nuclear Shadowing in QCD



Shadowing depends on understanding leading twist-diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF !

Dynamical effect due to virtual photon interacting in nucleus



Shadowing depends on leading-twist DDIS

Integration over on-shell domain produces phase i

Need Imaginary Phase to Generate Pomeron

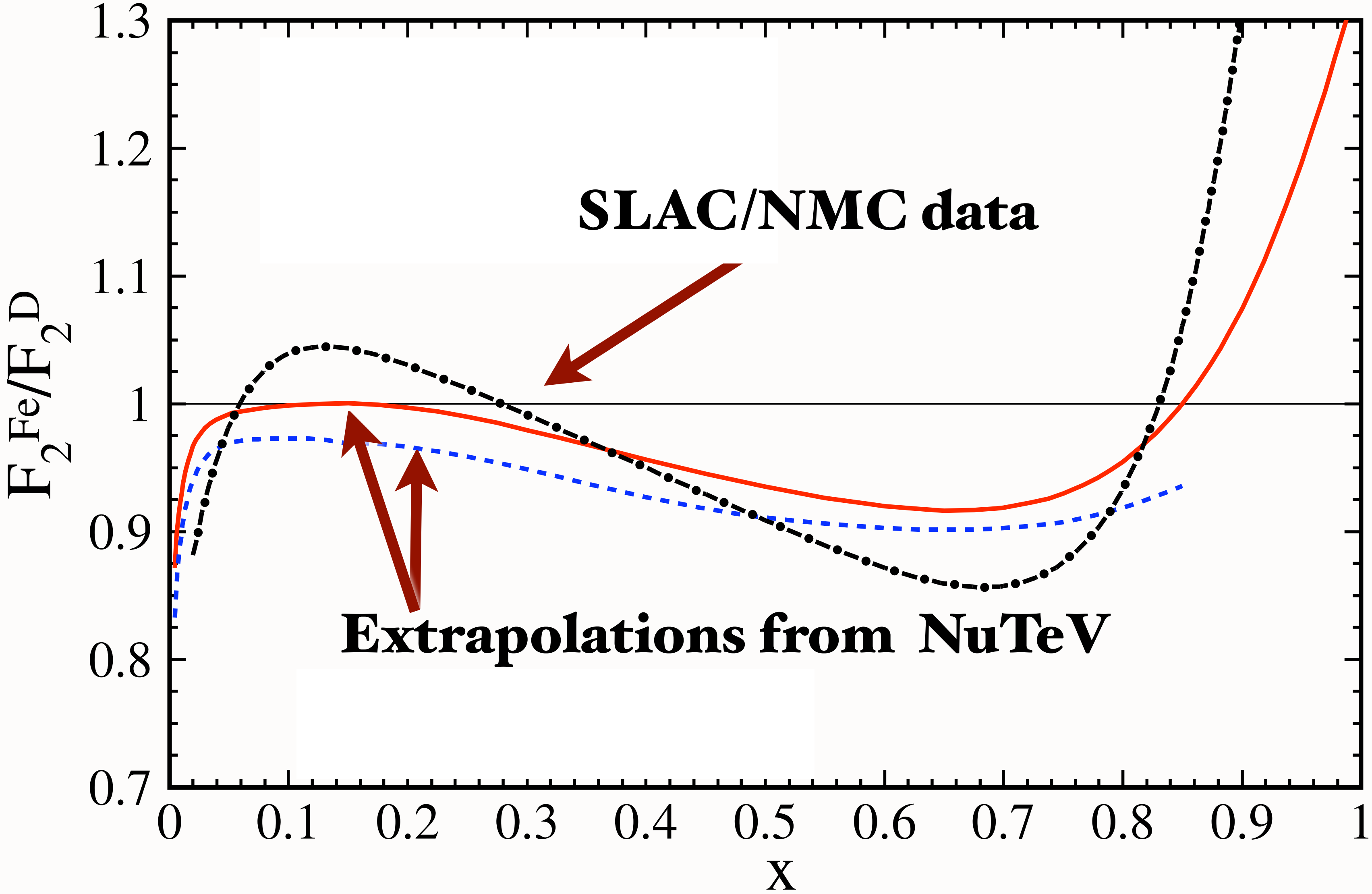
Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target

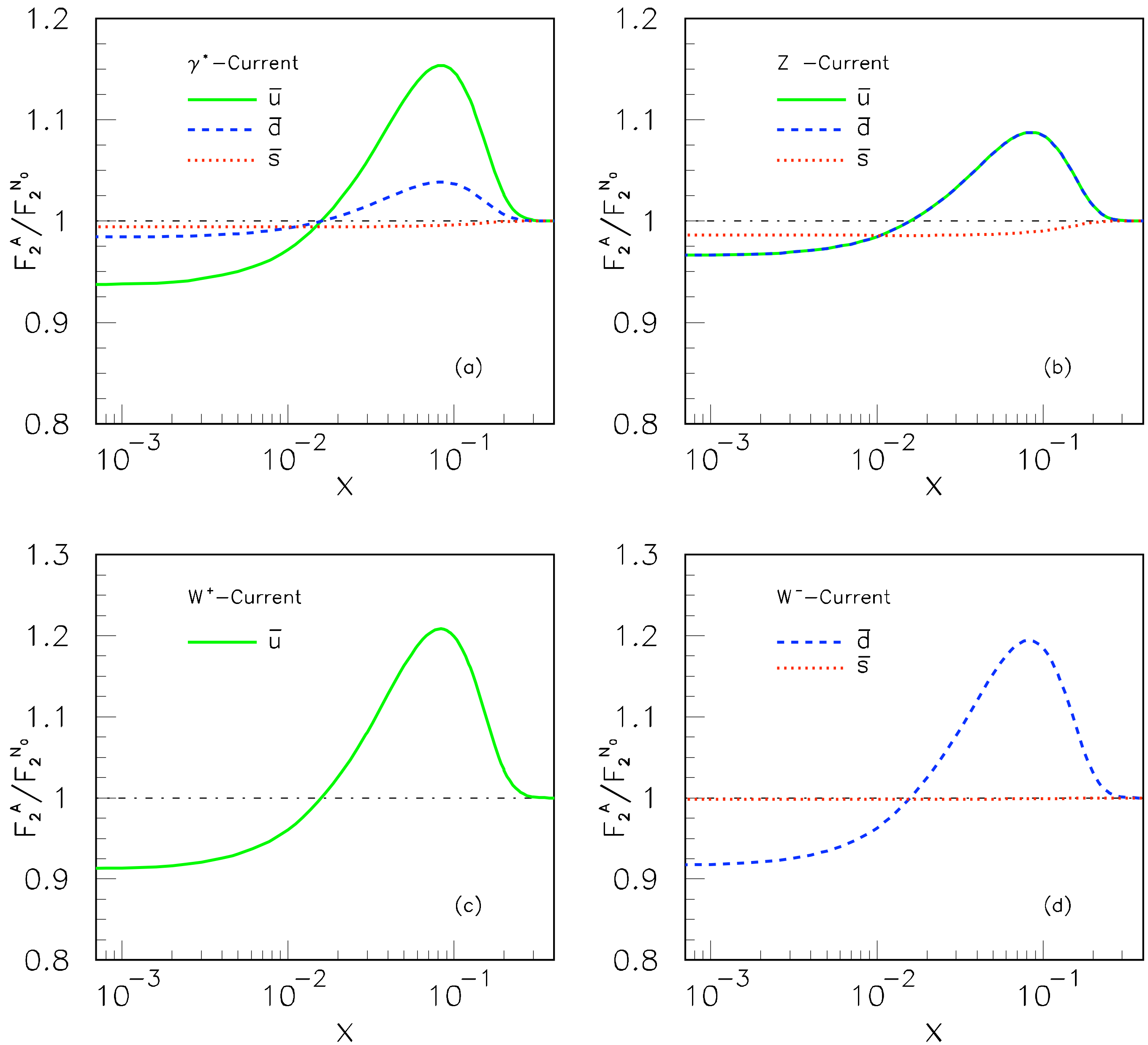
Antishadowing (Reggeon exchange) is not universal!

Schmidt, Yang, sjb

$$Q^2 = 5 \ \mathrm{GeV}^2$$



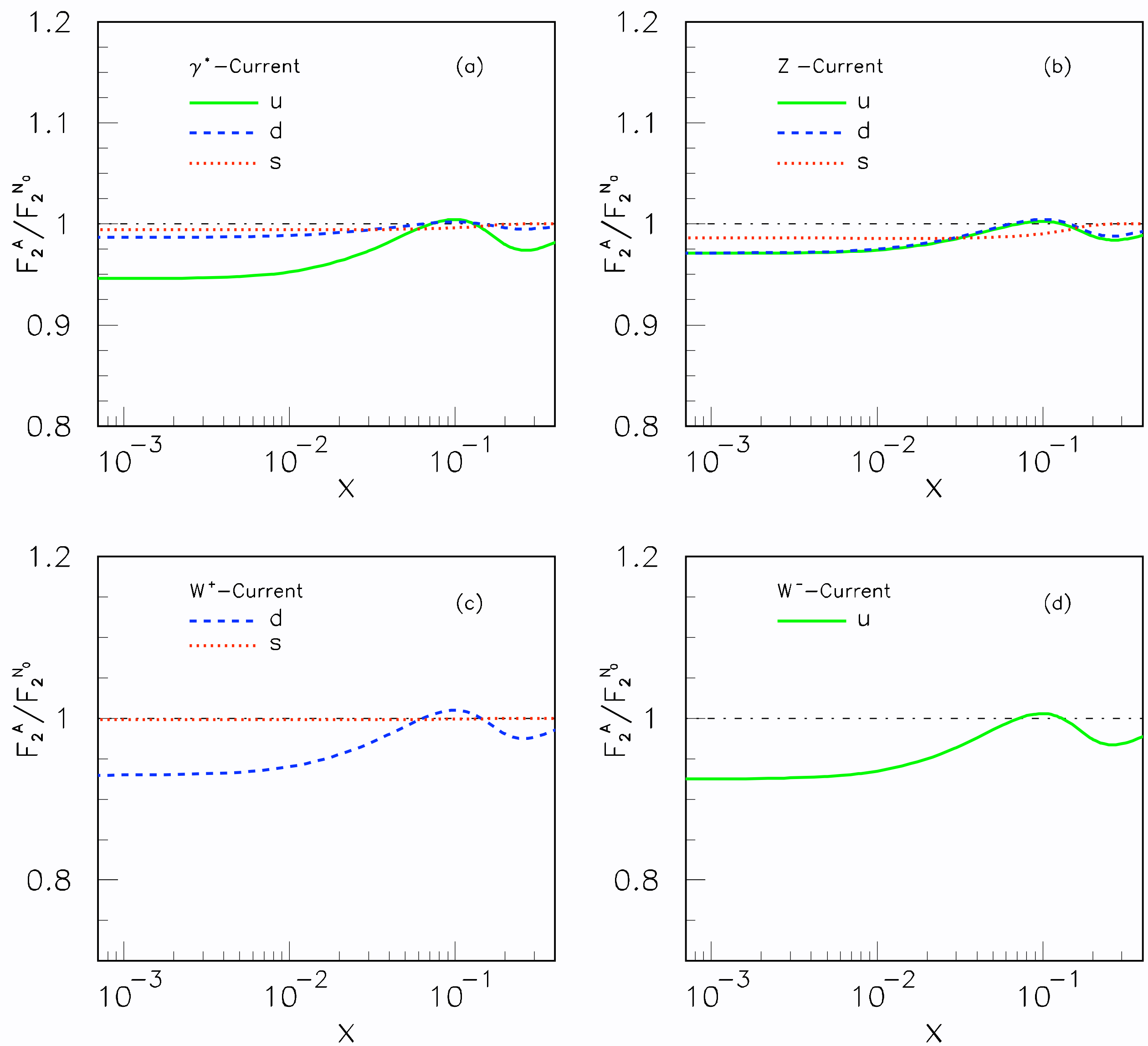
Scheinbein, Yu, Keppel, Morfin, Olness, Owens



Schmidt, Yang; sjb

Nuclear Antishadowing not universal !

Shadowing and Antishadowing of DIS Structure Functions

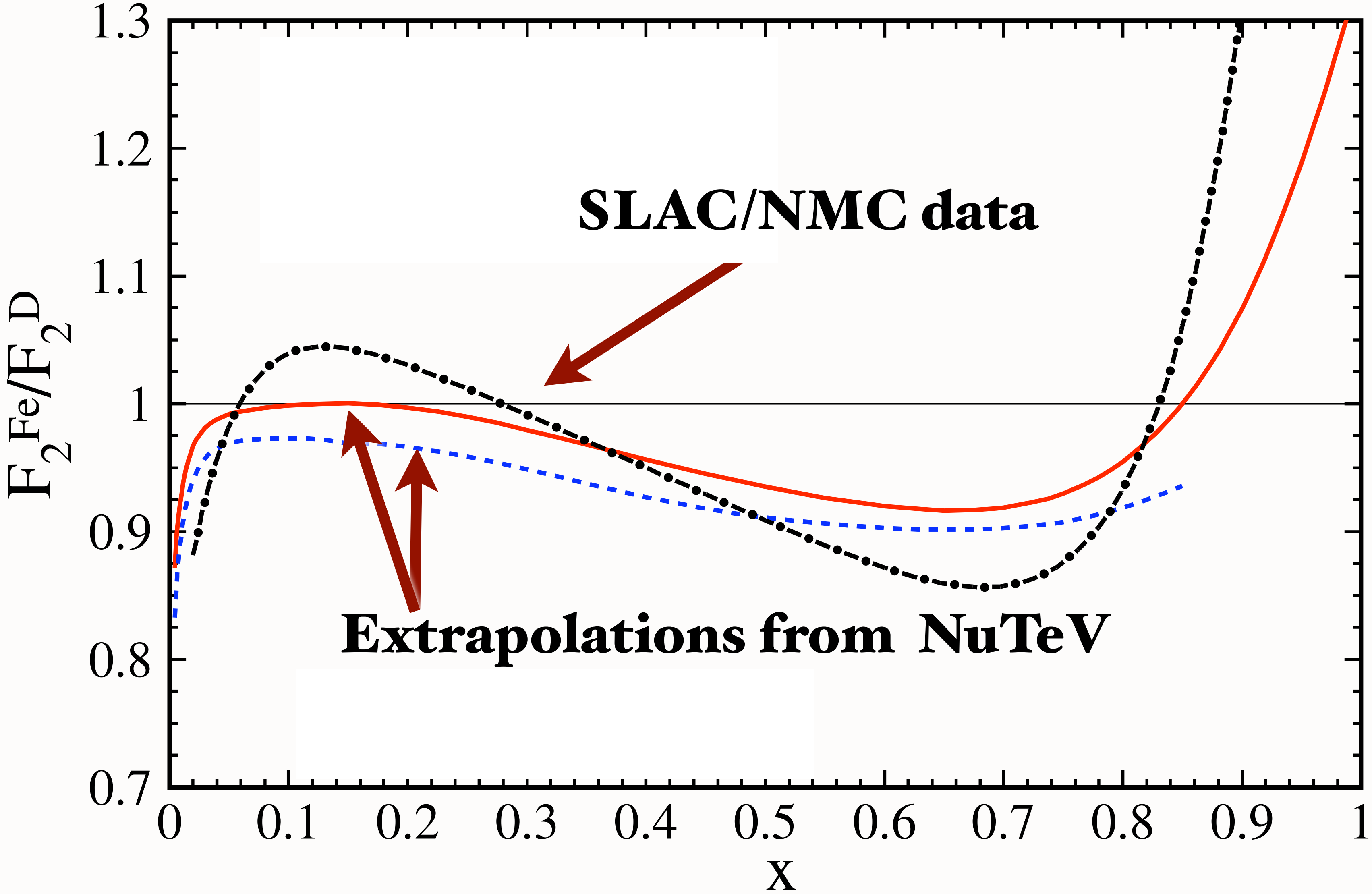


S. J. Brodsky, I. Schmidt and J. J. Yang, "Nuclear Antishadowing in Neutrino Deep Inelastic Scattering," Phys. Rev. D 70, 116003 (2004) [arXiv:hep-ph/0409279].

Modifies NuTeV extraction of $\sin^2\theta_W$

Test in flavor-tagged lepton-nucleus collisions

$$Q^2 = 5\ \mathrm{GeV}^2$$



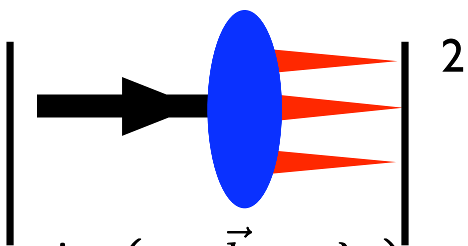
Scheinbein, Yu, Keppel, Morfin, Olness, Owens

Physics of Rescattering

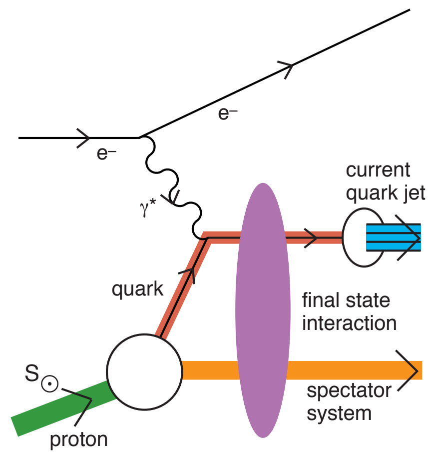
- Diffractive DIS
- Non-Unitary Correction to DIS: Structure functions are not probability distributions
- Nuclear Shadowing, Antishadowing- Not in Target WF
- Single Spin Asymmetries -- opposite sign in DY and DIS
- DY angular distribution at leading twist from double ISI-- not given by PQCD factorization -- breakdown of factorization!
- Wilson Line Effects not 1 even in LCG
- Must correct hard subprocesses for initial and final-state soft gluon attachments
- Corrections to Handbag Approximation in DVCS

Hoyer, Marchal, Peigne, Sannino, sjb

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

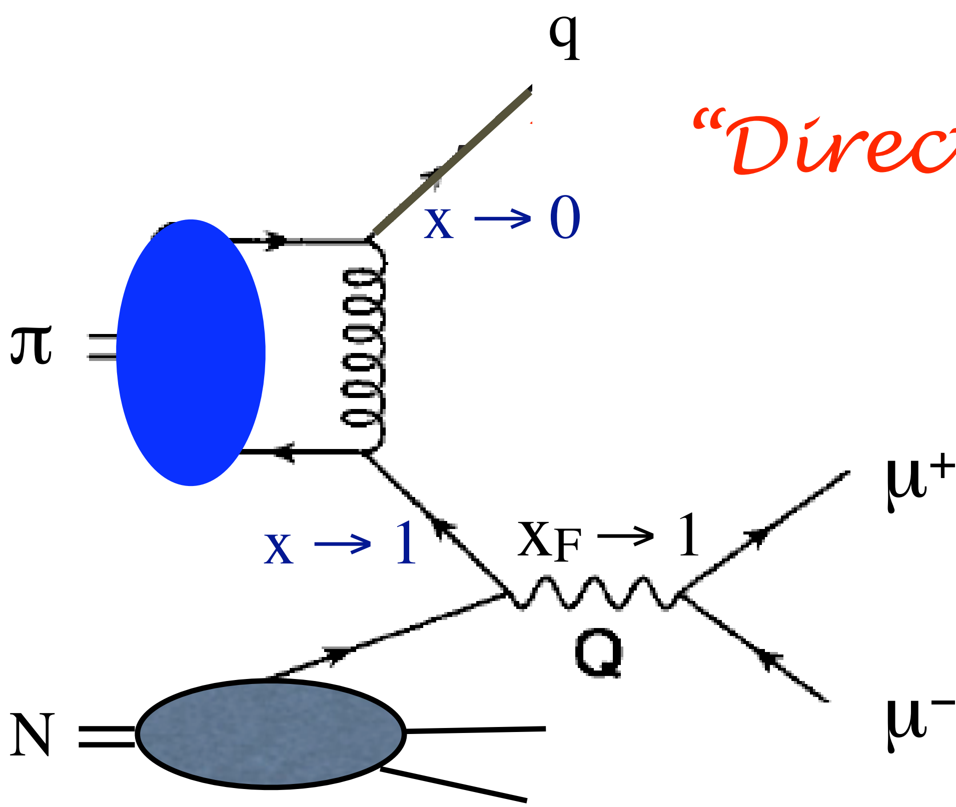


$$\left| \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2$$



$\pi N \to \mu^+ \mu^- X$ at high x_F

In the limit where $(1-x_F)Q^2$ is fixed as $Q^2 \to \infty$



"Direct" Subprocess

Entire pion wf contributes to hard process

Virtual photon is longitudinally polarized

Berger, sjb
Khoze, Brandenburg, Muller, sjb
Hoyer Vanttinen

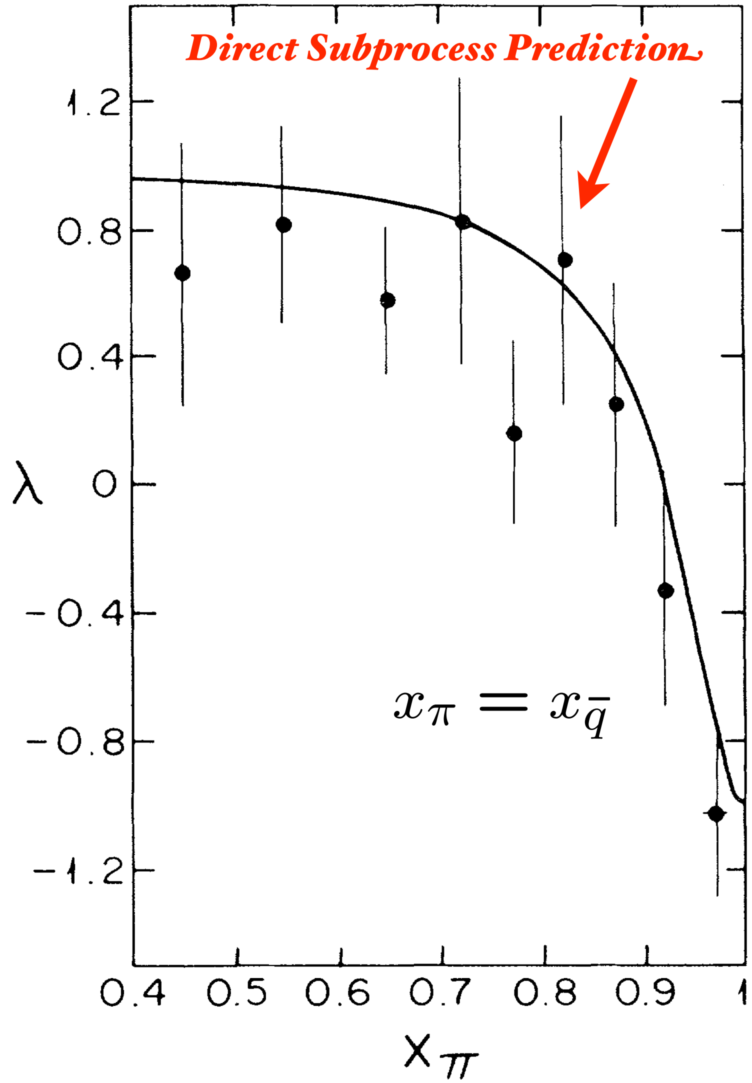
$\pi^- N \to \mu^+ \mu^- X$ at 80 GeV/c

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi .$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1 - x_\pi)^2 (1 + \cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \ \mathrm{GeV}^2/c^2$$

Dramatic change in angular distribution at large x_F

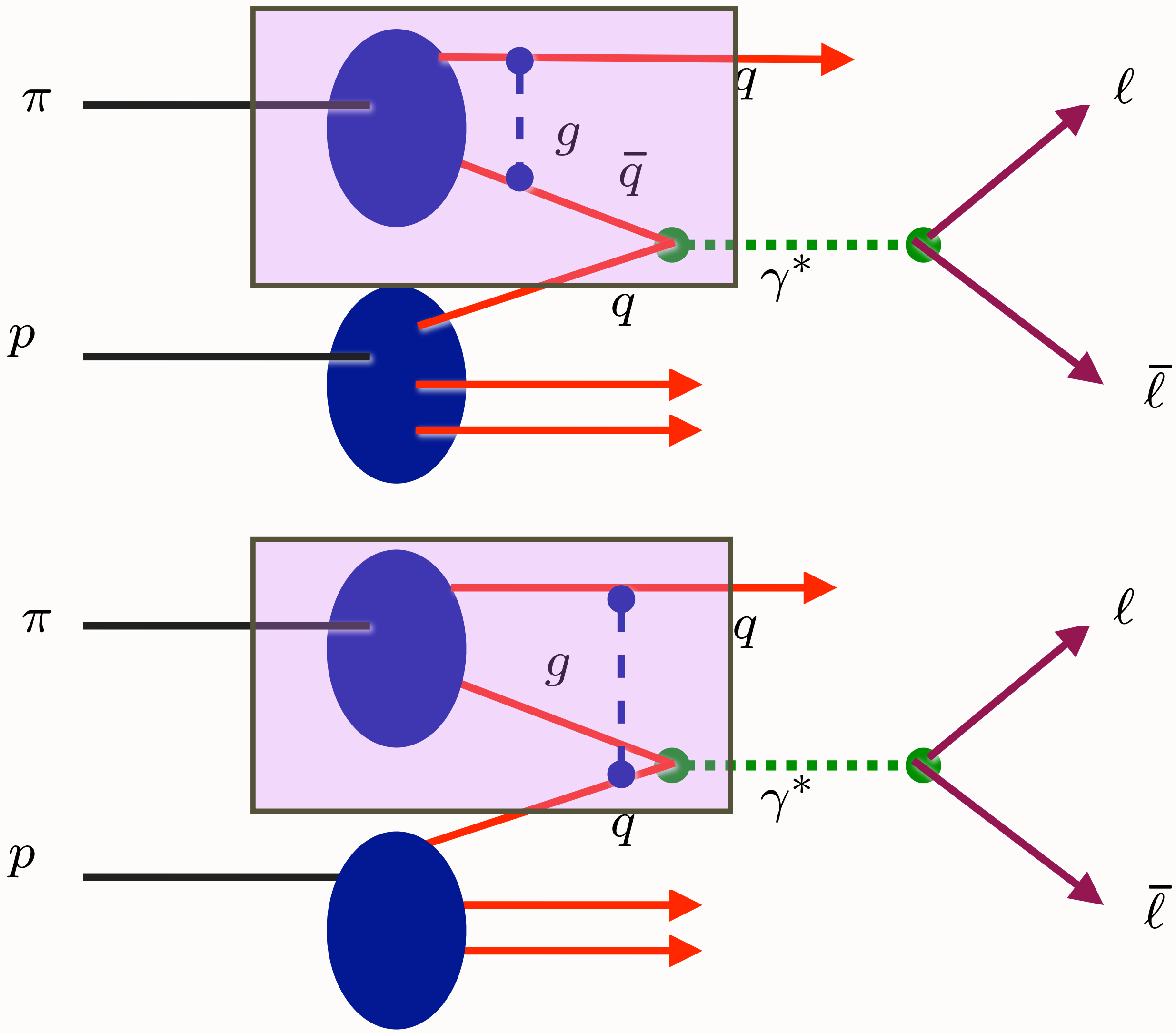


Chicago-Princeton Collaboration

Phys.Rev.Lett.55:2649,1985

Example of a higher-twist direct subprocess

Berger, Lepage, sjb



$$\pi q \to \gamma^* q$$

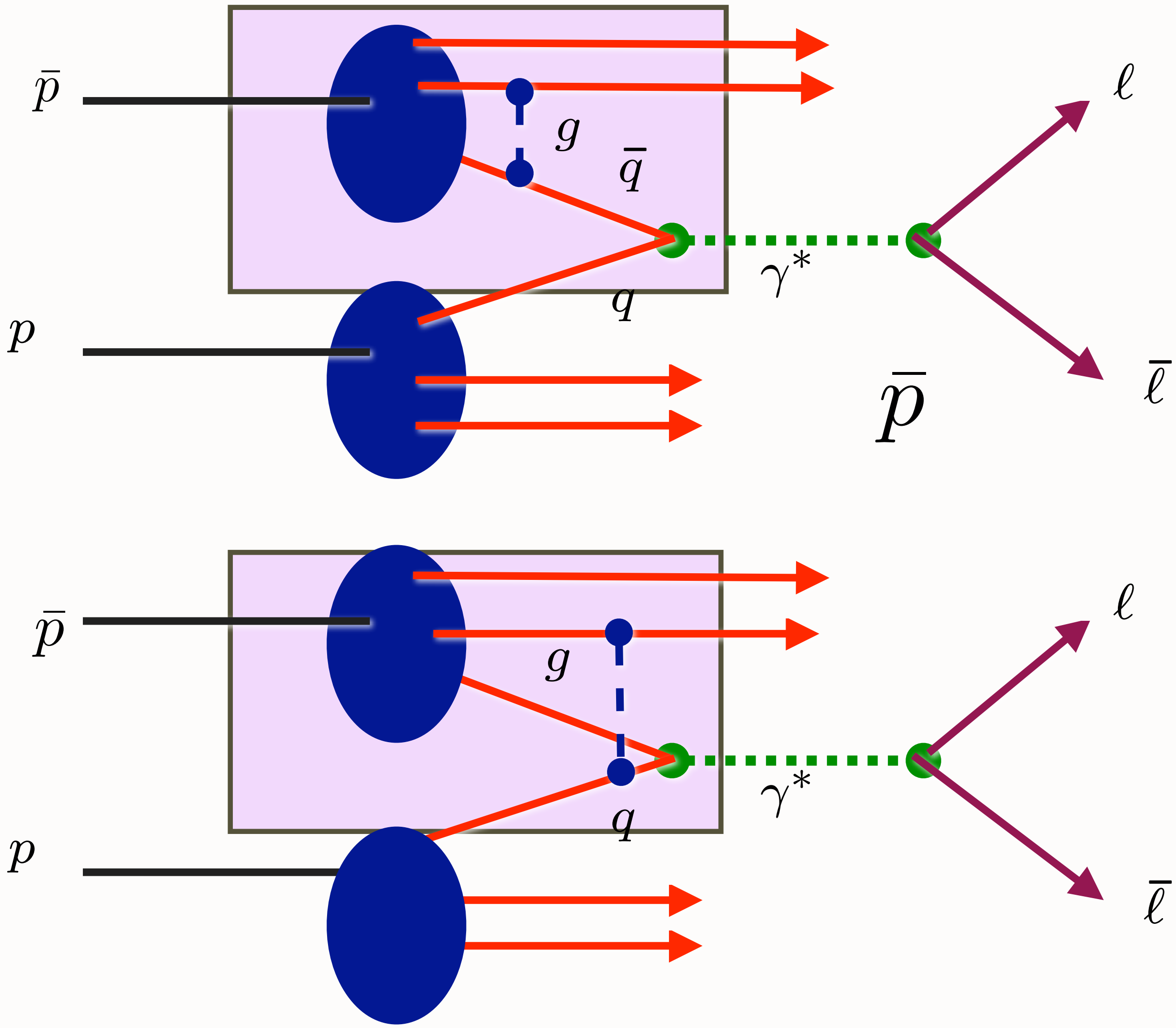
Initial State Interaction

Pion appears directly in subprocess at large x_F

All of the pion's momentum is transferred to the lepton pair

Lepton Pair is produced longitudinally polarized

$$A(1-x)^3(1+\cos^2\theta) + B\frac{(1-x)\sin^2\theta}{Q^2} + C\frac{(1+\cos^2\theta)}{(1-x)Q^4}$$



$$[\bar{q}\bar{q}]q \to \gamma^* \bar{q}$$

Diquark appears directly in subprocess medium x_F

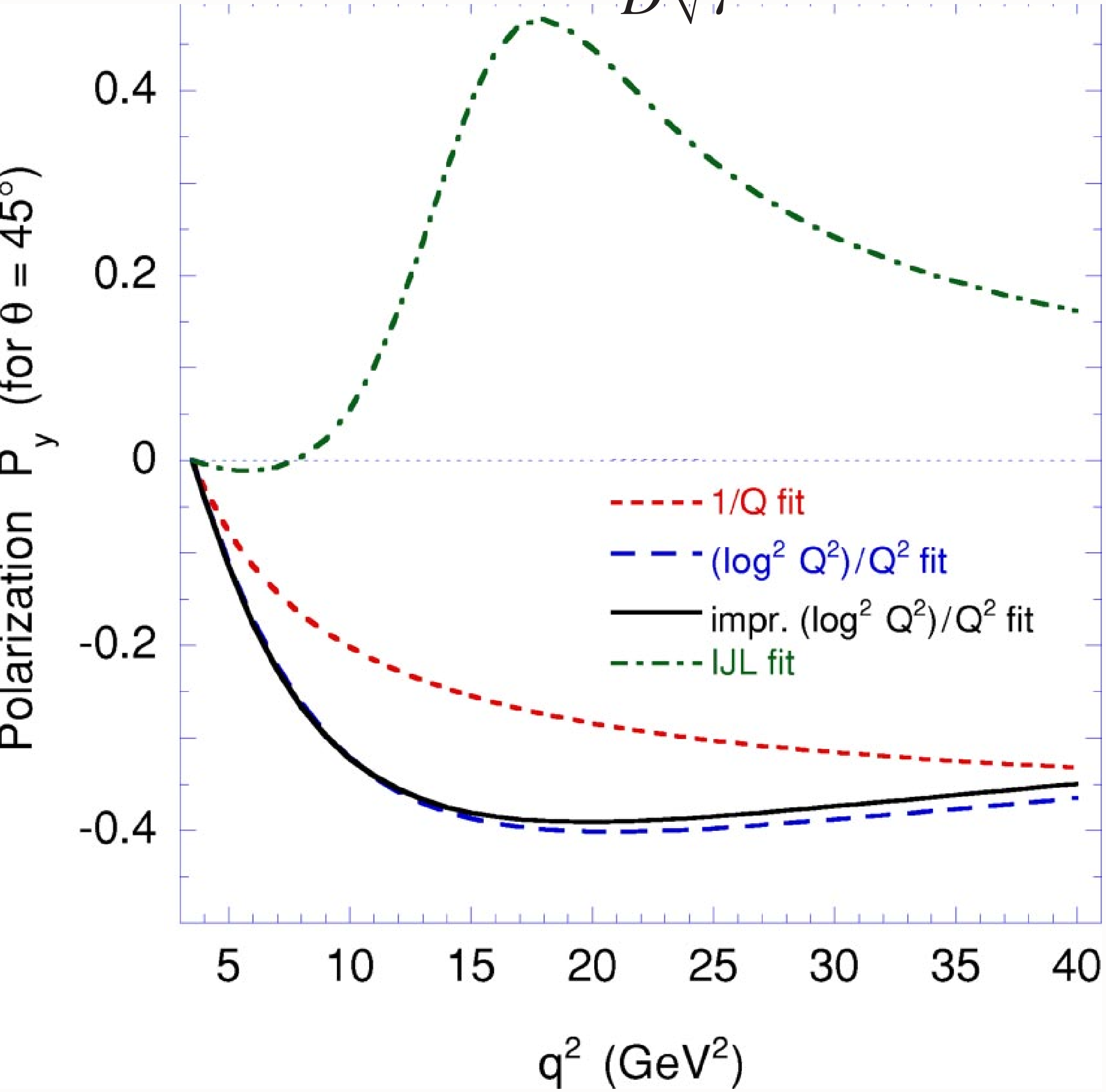
All of the diquark's momentum is transferred to the lepton pair

Lepton Pair is produced longitudinally polarized

Single-spin polarization effects and the determination of timelike proton form factors

$$\mathcal{P}_y = \frac{\sin 2\theta \,\mathrm{Im}\, G_E^* G_M}{D\sqrt{\tau}} = \frac{(\tau-1)\sin 2\theta \,\mathrm{Im}\, F_2^* F_1}{D\sqrt{\tau}}$$

Carlson, Hiller, Hwang, sjb



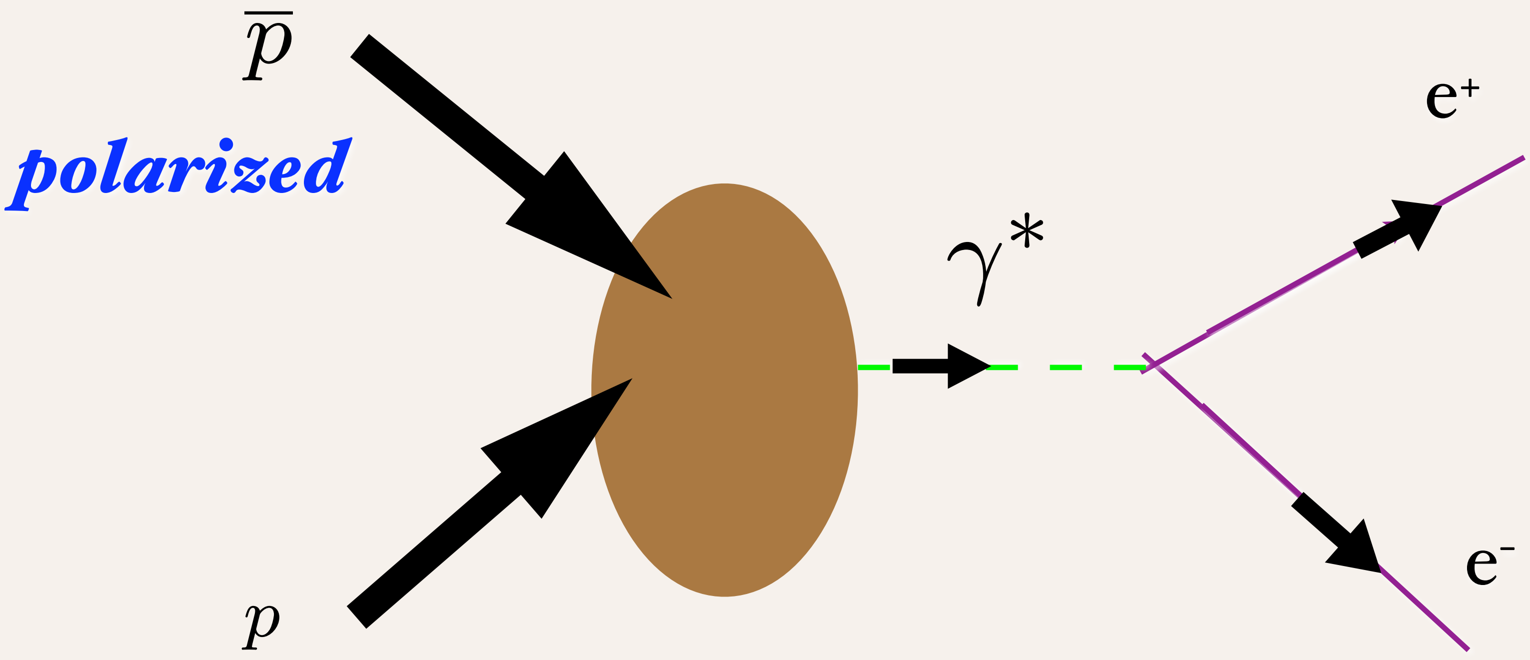
$$D = |G_M|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E|^2\sin^2\theta;$$

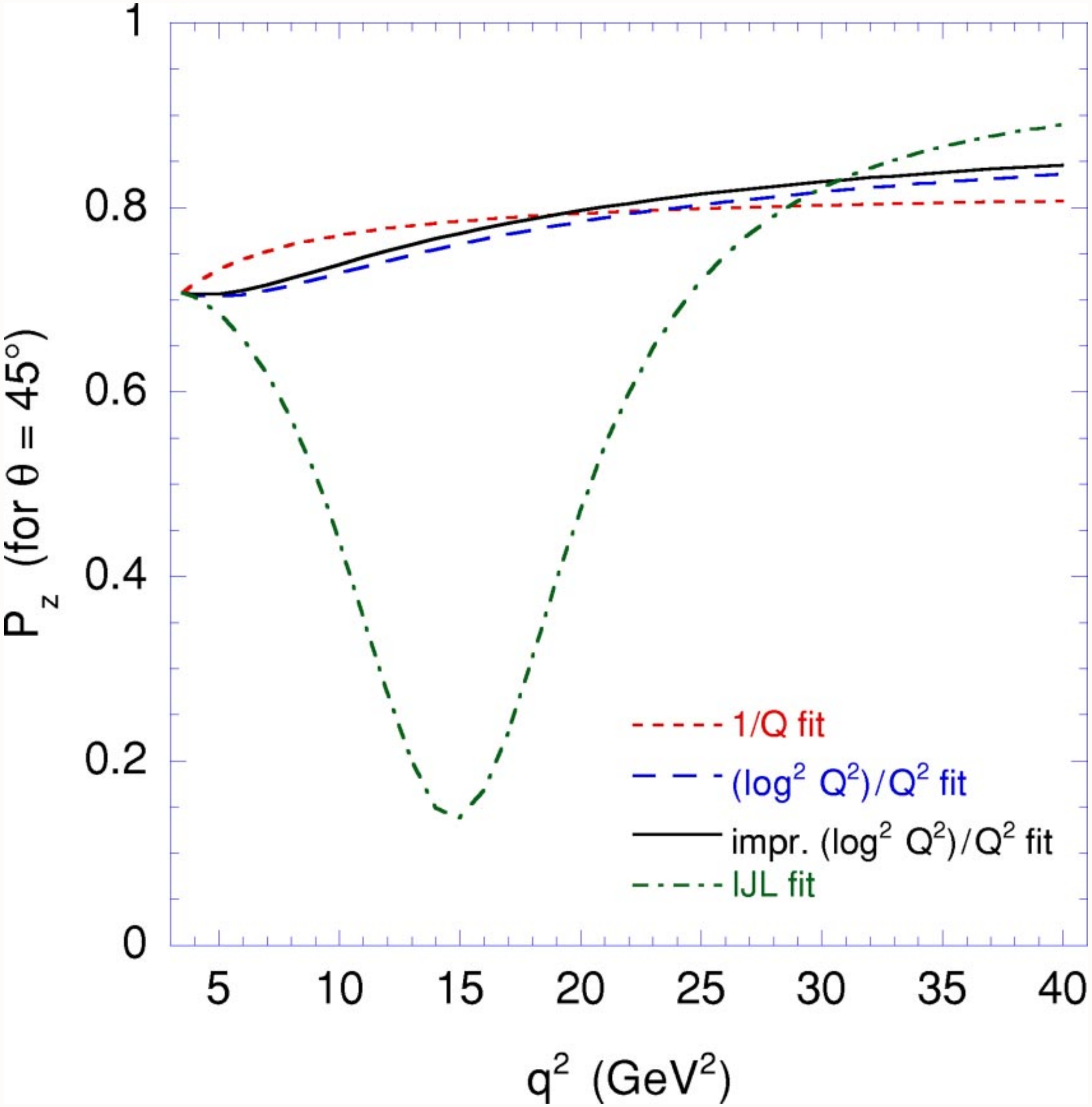
$$\tau \equiv q^2/4m_B^2$$

Measure relative phase of form factors

Key QCD Experiment at FAIR

$$\mathcal{P}_y = \frac{\sin 2\theta \,\mathrm{Im}\, G_E^* G_M}{D\sqrt{\tau}} = \frac{(\tau - 1)\sin 2\theta \,\mathrm{Im}\, F_2^* F_1}{D\sqrt{\tau}}$$





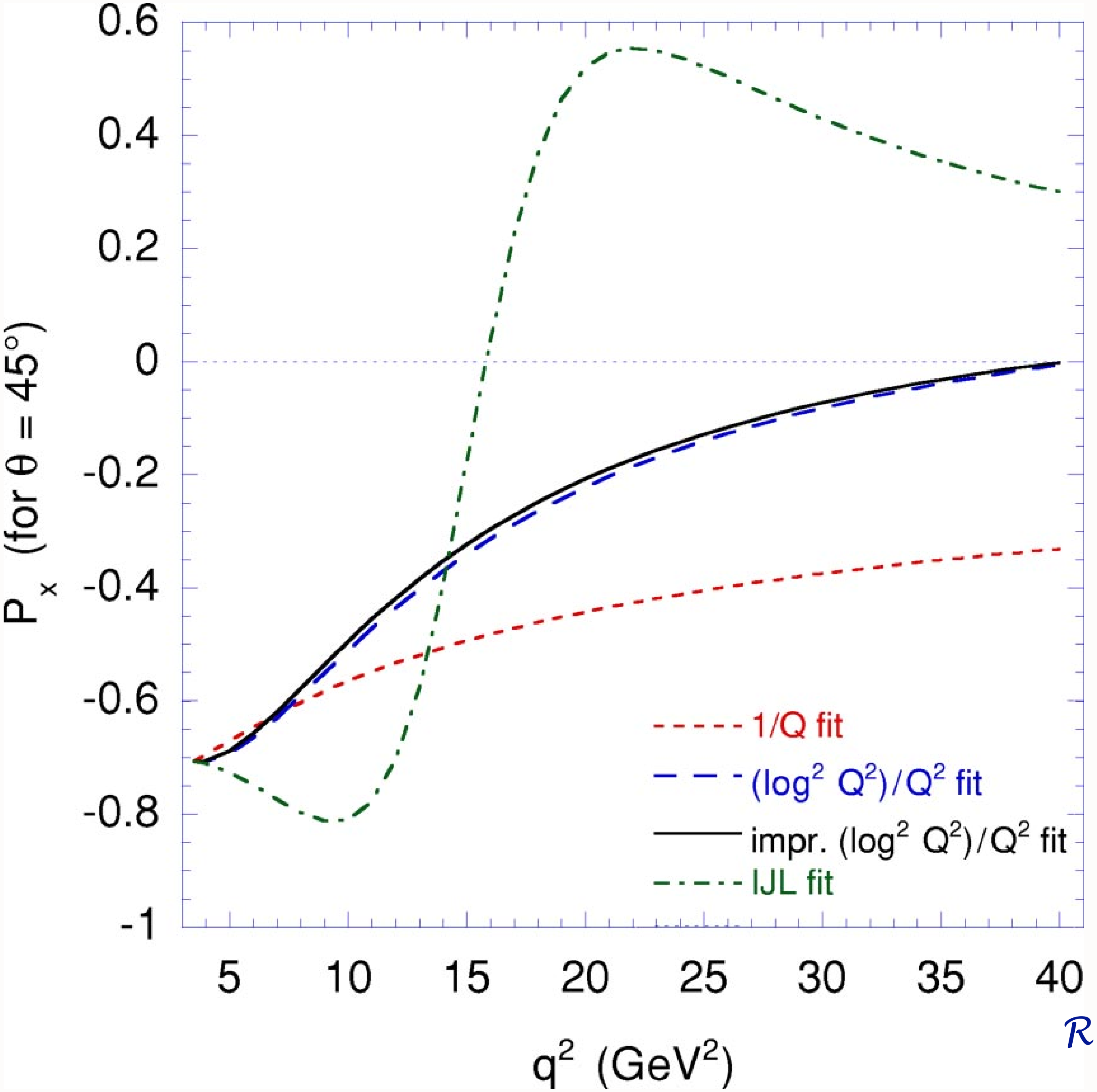
Carlson, Hiller, Hwang, sjb

$$\mathcal{P}_z = P_e \frac{2\cos\theta |G_M|^2}{D}$$

$$D = |G_M|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E|^2 \sin^2\theta;$$

Requires beam and lepton polarization

Single-spin polarization effects and the determination of timelike proton form factors



Carlson, Hiller, Hwang, sjb

$$\mathcal{P}_x = -P_e \frac{2\sin\theta \operatorname{Re} G_E^* G_M}{D\sqrt{\tau}}$$

$$D = |G_M|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E|^2 \sin^2\theta;$$

Requires beam and lepton polarization

Heavy Quark Anomalies

$J/\psi \to \rho\pi$ puzzle

BR = 1.27 +/- 0.09 %

Largest two-body decay channel

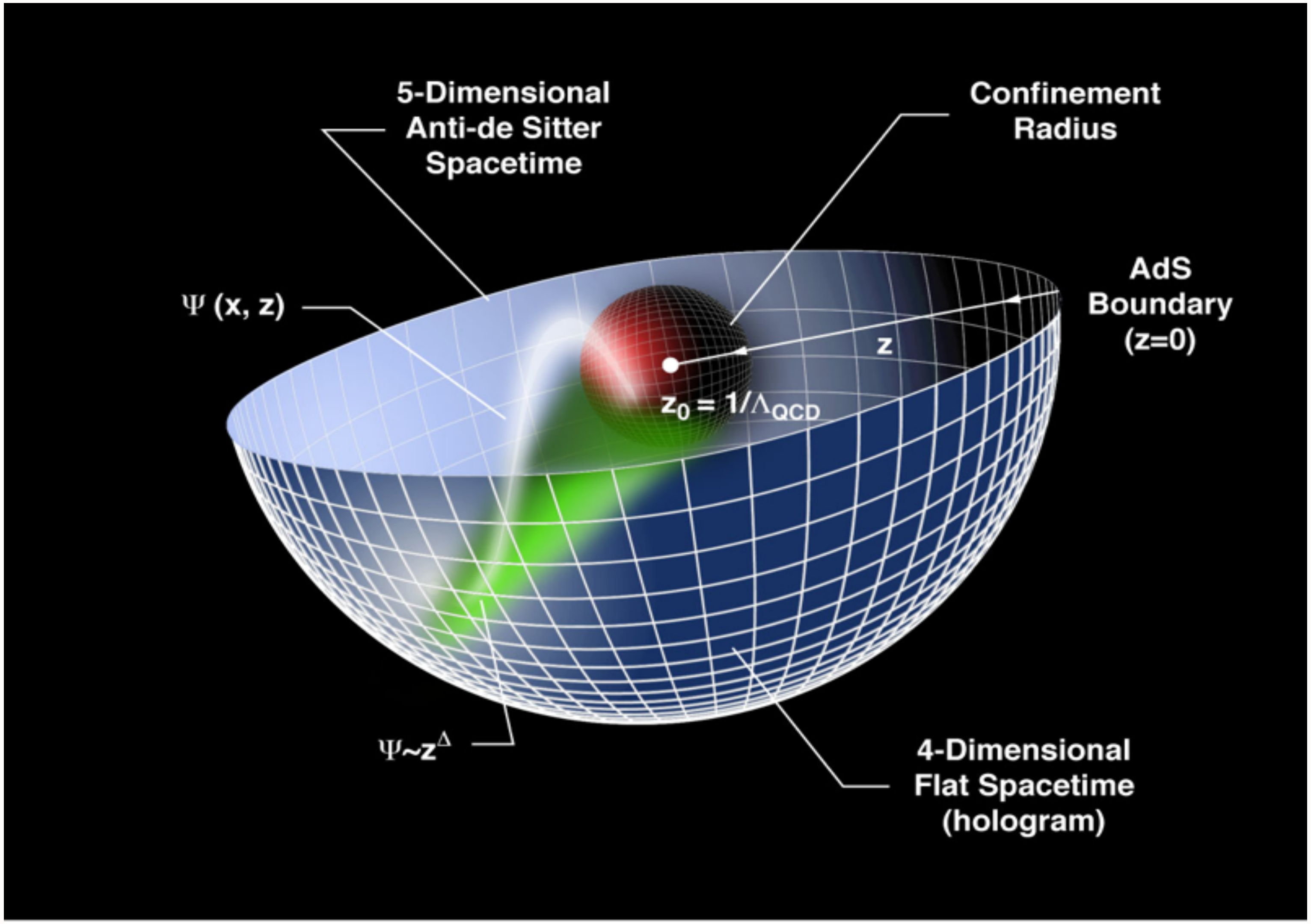
Violates hadron helicity conservation

ψ' almost never decays to $\rho\ \pi$ $< 8.3 \times 10^{-5}$

Solution: Intrinsic charm Fock states in ρ, π

Karliner, sjb

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathrm{M}^{\mu\nu}, \mathrm{P}^{\mu}, \mathrm{D}, \mathrm{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu,\ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

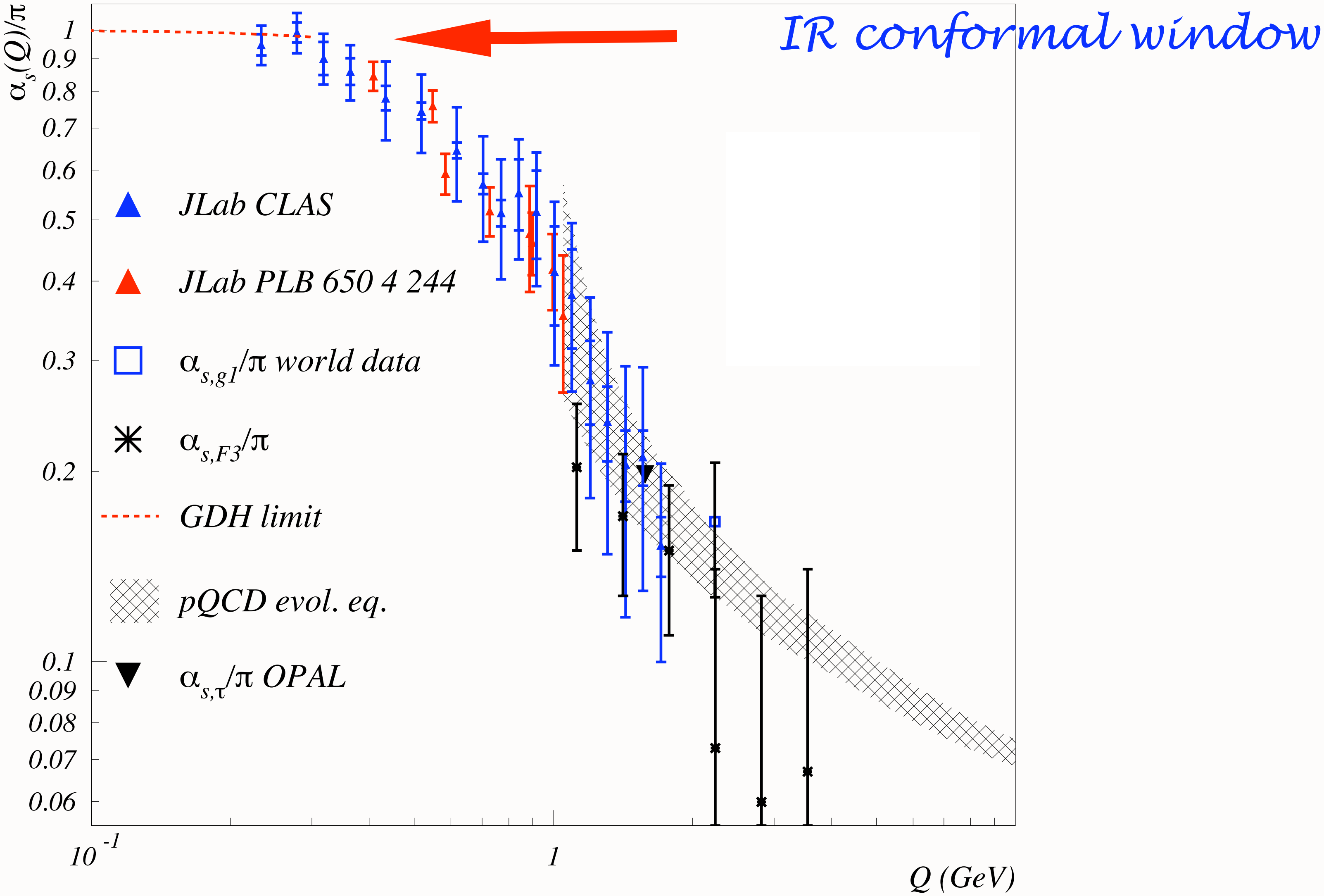
$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

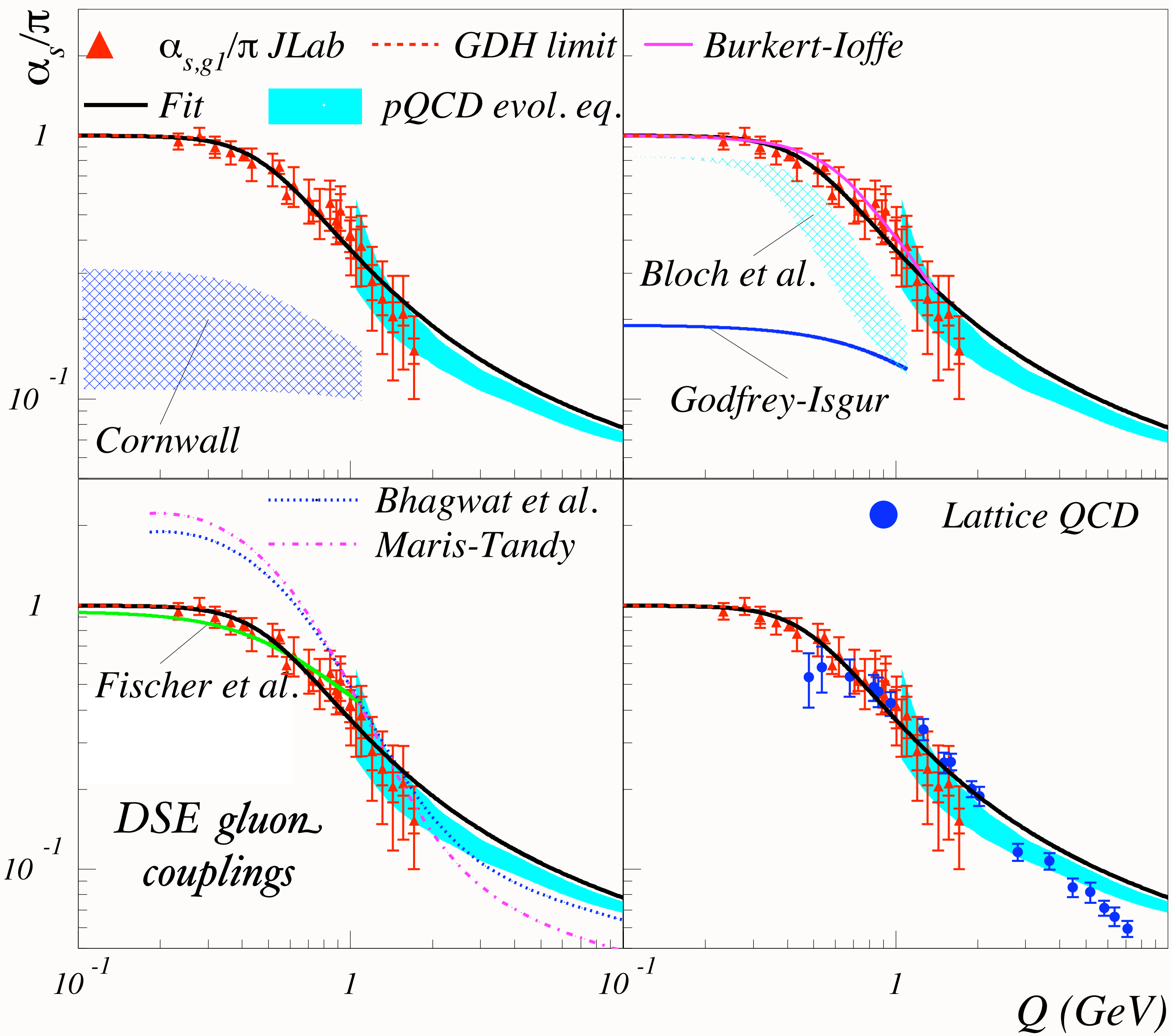
- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}]$$



Deur, Korsch, et al.



IR Conformal Window for QCD?

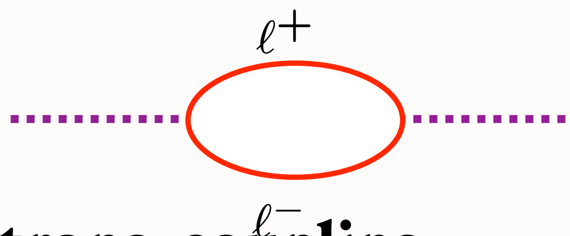
- *Dyson-Schwinger Analysis:* **QCD gluon coupling has IR Fixed Point**

- *Evidence from Lattice Gauge Theory*

- Define coupling from observable: **indications of IR fixed point for QCD effective charges** Shrock, de Teramond, sjb

- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small Q^2**

$$\Pi(Q^2) \to \frac{\alpha}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4m^2$$



- **Justifies application of AdS/CFT in strong-coupling conformal window**

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map AdS_5 X S_5 to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$

- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

String Theory

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space
to AdS5 space

AdS/CFT

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories

QCD at the Amplitude Level

Conformal behavior at short distances
+ Confinement at large distance

AdS/QCD

Semi-Classical QCD / Wave Equations

Holography

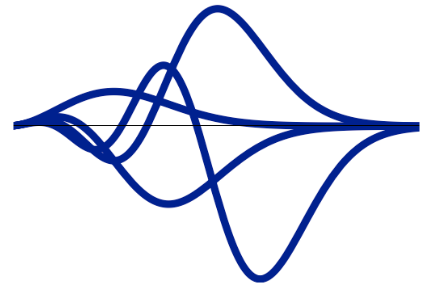
Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Soft-Wall Model



- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce smooth cutoff wich depends on the profile of a dilaton background field $\varphi(z) = \pm\kappa^2 z^2$

$$S = \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L},$$

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2}\left(g^{\ell m}\partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2\right)$

$$\left[z^2 \partial_z^2 - \left(d - 1 \mp 2\kappa^2 z^2\right) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$. See also [Metsaev (2002), Andreev (2006)]

- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 4\kappa^2 n, \quad \Phi_n(z) \sim z^2 e^{-\kappa^2 z^2} L_n(\kappa^2 z^2)$$

$\Phi_0(z)$ a chiral symmetric bound state of two massless quarks with scaling dimension 2: the pion

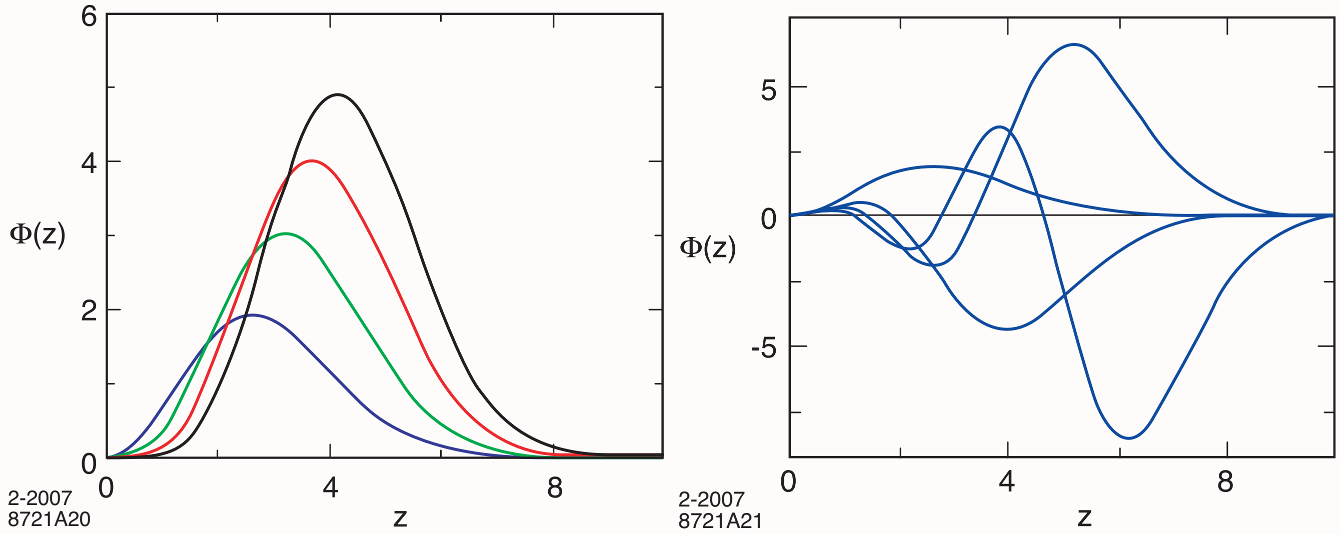
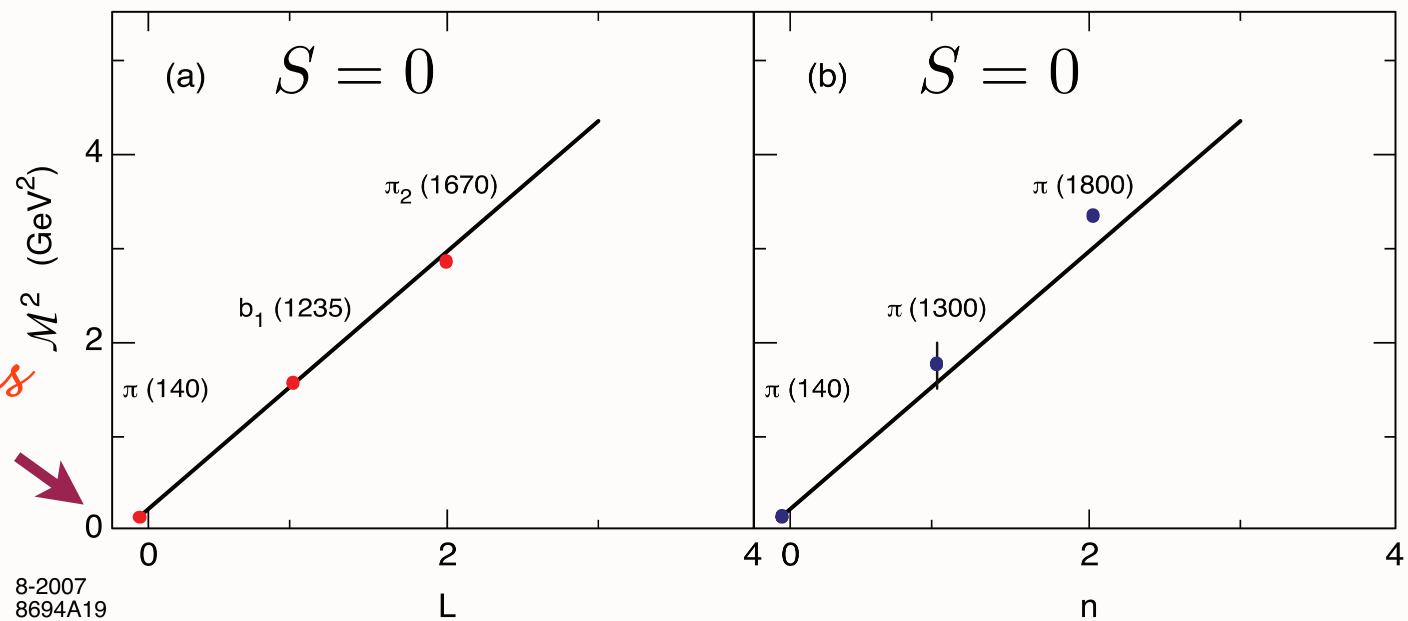


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model



Pion has zero mass!

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Soft-wall model

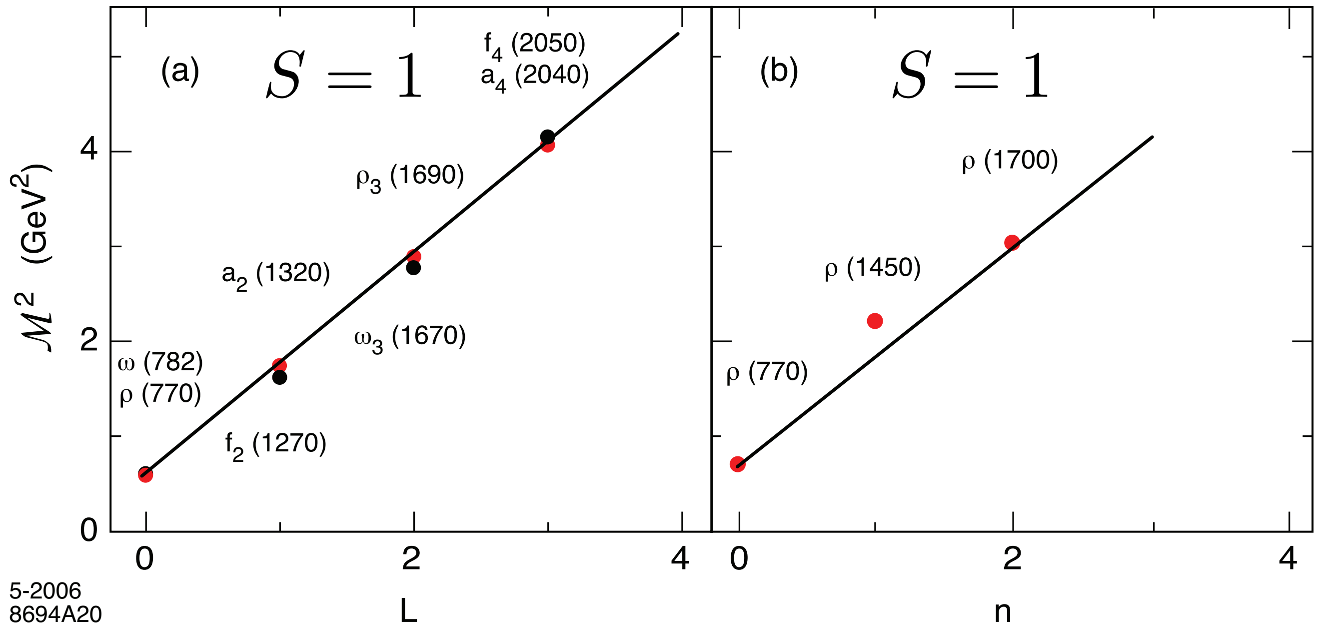
- Effective LF Schrödinger wave equation

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L+S-1)\right]\phi_S(z) = \mathcal{M}^2 \phi_S(z)$$

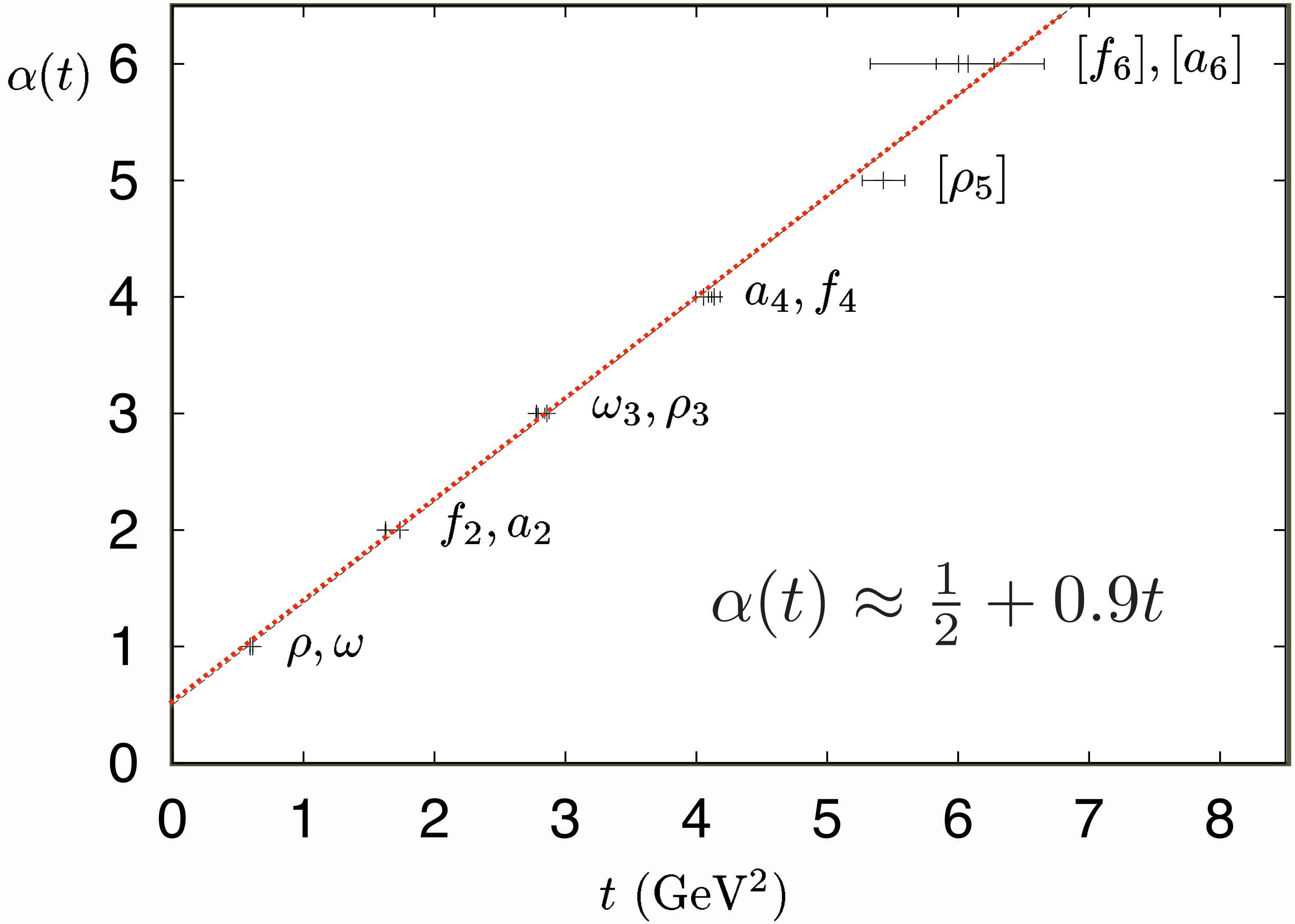
with eigenvalues $\mathcal{M}^2 = 2\kappa^2(2n + 2L + S)$.

Same slope in n and L

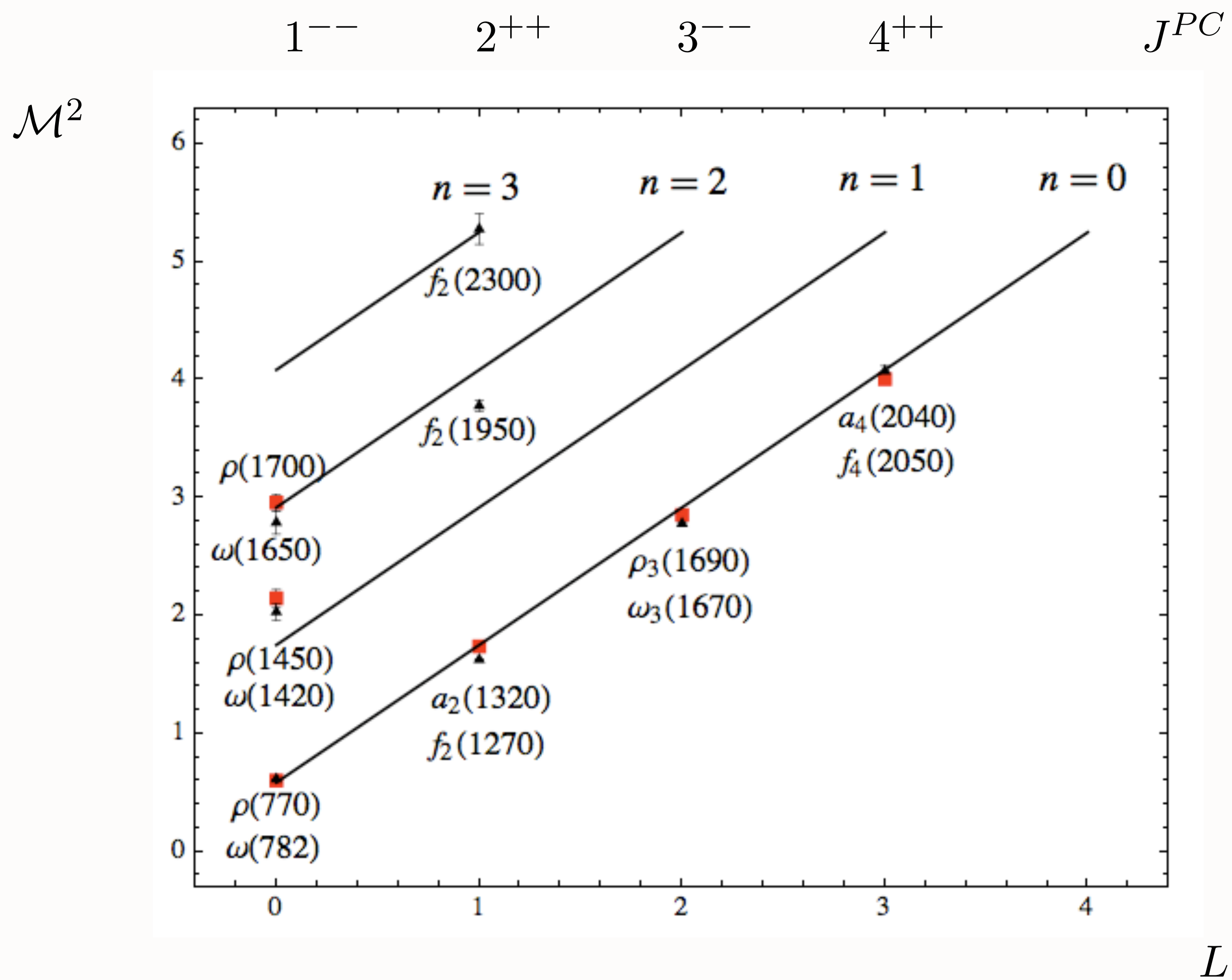
- Compare with Nambu string result (rotating flux tube): $M_n^2(L) = 2\pi\sigma\,(n + L + 1/2)$.



Vector mesons orbital (a) and radial (b) spectrum for $\kappa = 0.54$ GeV.

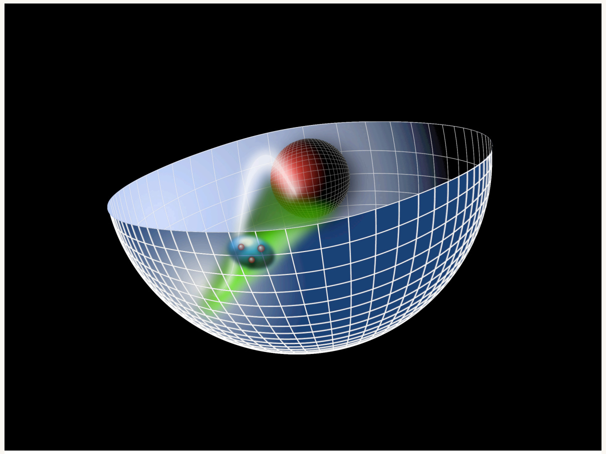


AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

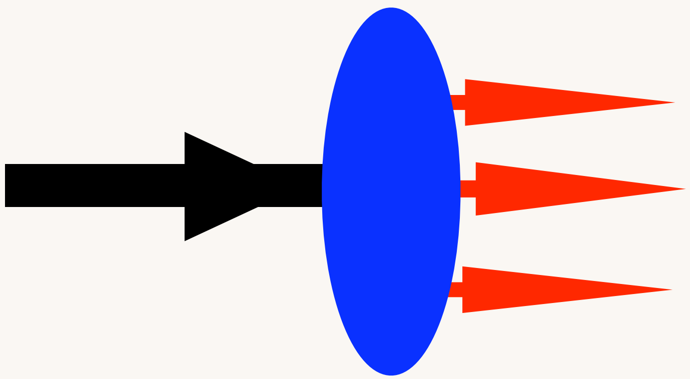


Parent and daughter Regge trajectories for the $I = 1$ ρ-meson family (red) and the $I = 0$ ω-meson family (black) for $\kappa = 0.54$ GeV

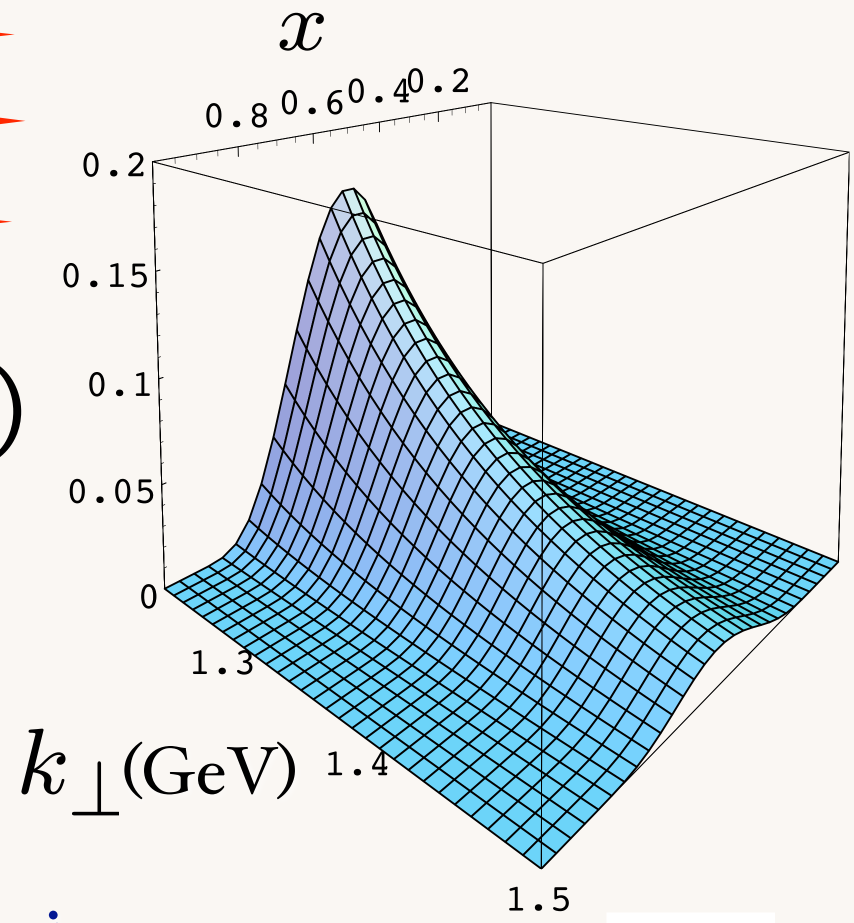
$\phi(z)$



- *Light-Front Holography*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



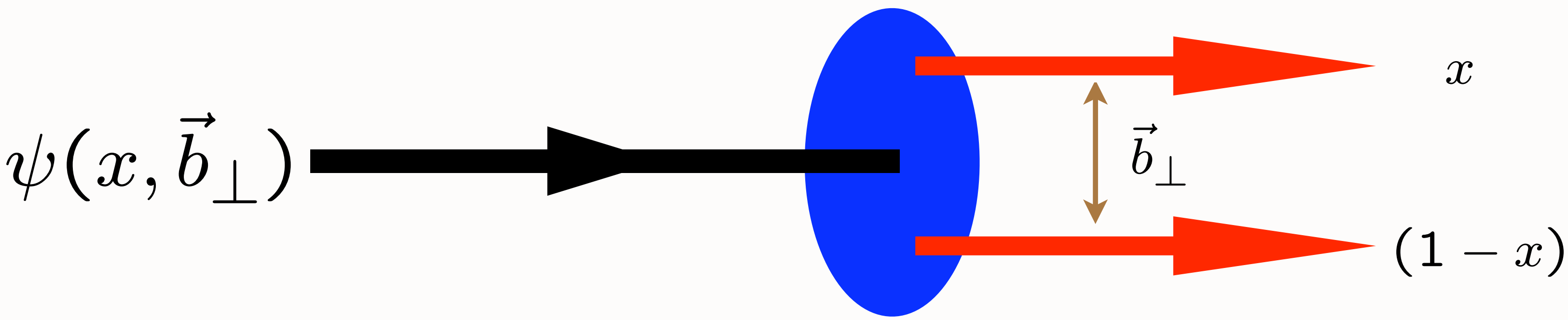
- *Light Front Wavefunctions:*

  Schrödinger Wavefunctions of Hadron Physics

LF(3+1) $\qquad\qquad$ AdS_5

$$\psi(x, \vec{b}_\perp) \qquad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \qquad z$$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

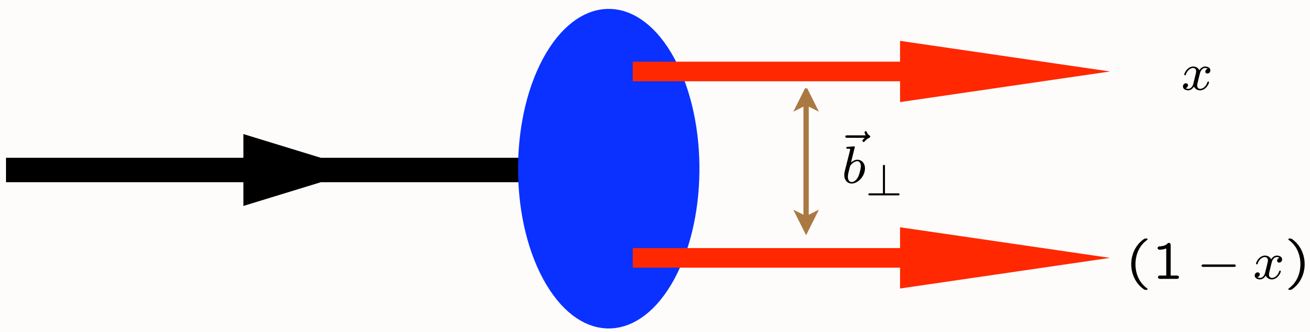
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

soft wall confining potential:

G. de Teramond, sjb

- Upon substitution $z \to \zeta$ $(J_z = L_z + S_z)$ we find for $d = 4$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2\zeta^2/2}\,\Phi_J(\zeta), \qquad (\mu R)^2 = -(2-J)^2 + L^2$$

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(L+S-1)\right)\phi_{\mu_1\cdots\mu_J} = \mathcal{M}^2\phi_{\mu_1\cdots\mu_J}}$$

- Eigenfunctions

$$\phi_{nL}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

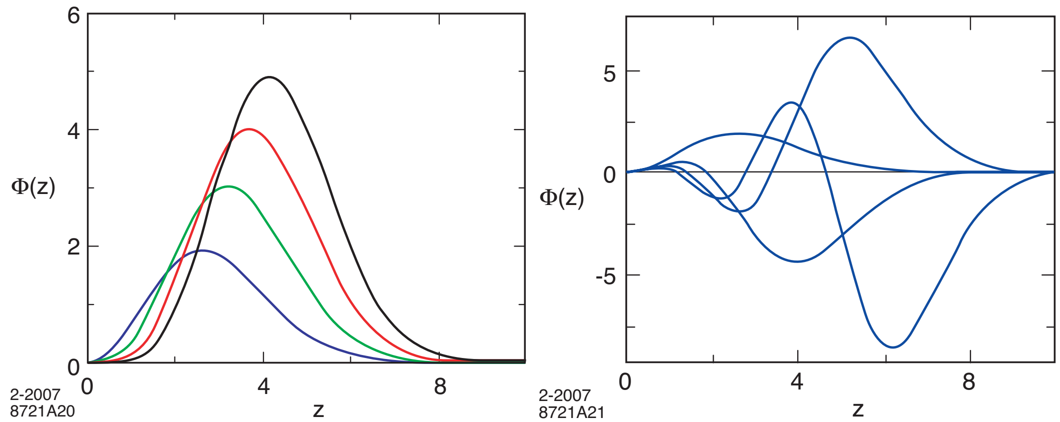
- Eigenvalues

$$\mathcal{M}^2_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2}\right)$$

$4\kappa^2$ for $\Delta n = 1$

$4\kappa^2$ for $\Delta L = 1$

$2\kappa^2$ for $\Delta S = 1$



Orbital and radial states: $\langle\zeta\rangle$ increase with L and n

- Ultra relativistic limit $m_q \to 0$ longitudinal modes decouple and LF eigenvalue equation $H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is a LF wave equation for ϕ

$$\boxed{\Big(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement}\Big)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)}$$

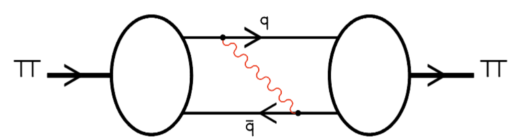
- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal light-front time

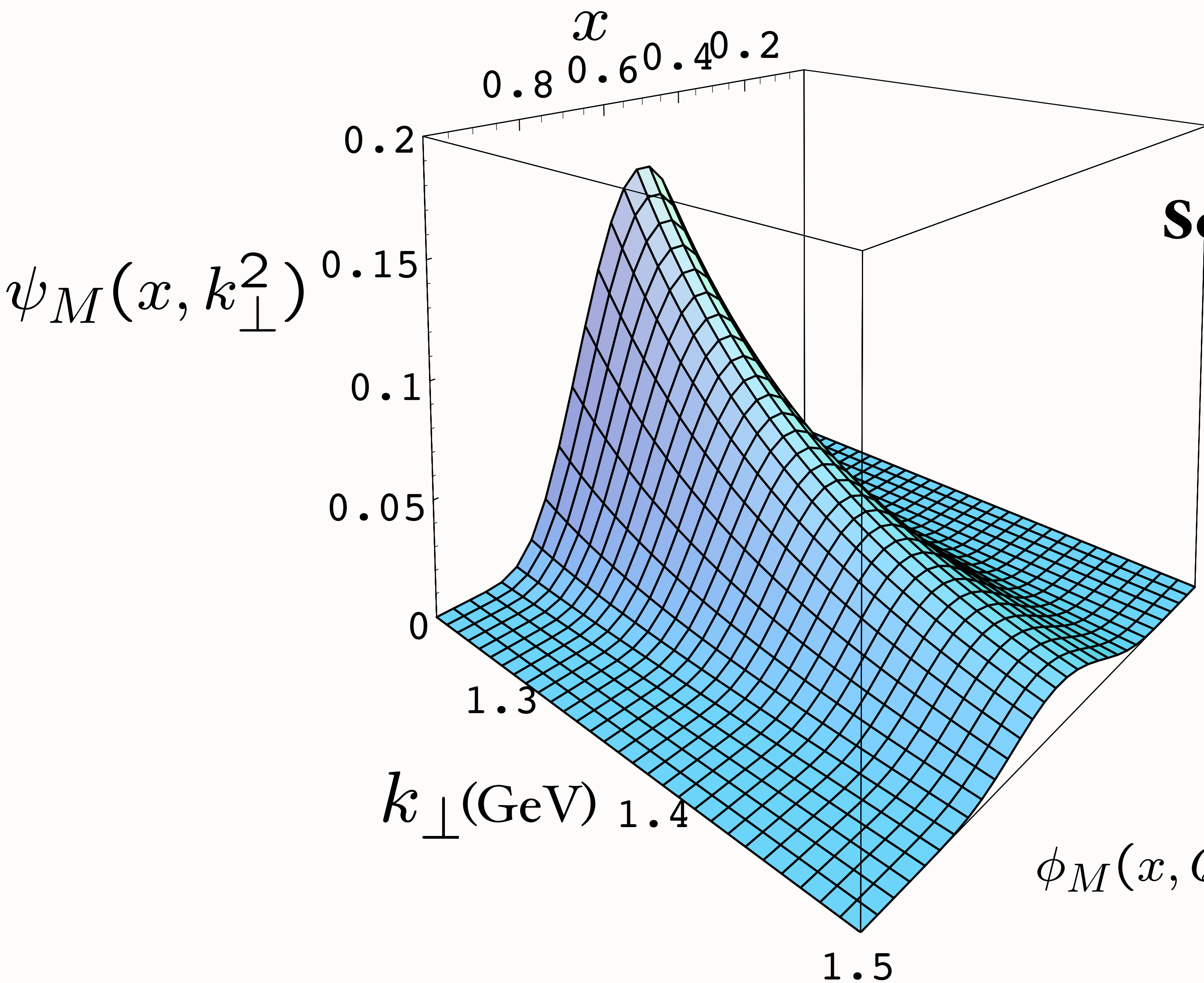
- LF modes $\phi(\zeta) = \langle\zeta|\phi\rangle$ are normalized by

$$\langle\phi|\phi\rangle = \int d\zeta\, |\langle\zeta|\phi\rangle|^2 = 1$$

- Semiclassical approximation to light-front QCD
  does not account for particle creation and absorption
  but can be implemented in the LF Hamiltonian EOM



Prediction from AdS/CFT: Meson LFWF



Soft Wall Model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Increases PQCD prediction for $F_\pi(Q^2)$ by 16/9

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

$$\text{Lattice (I)} \; < \xi^2 >_\pi = 0.28 \pm 0.03$$ Donnellan et al.

$$\text{Lattice (II)} \; < \xi^2 >_\pi = 0.269 \pm 0.039$$ Braun et al.

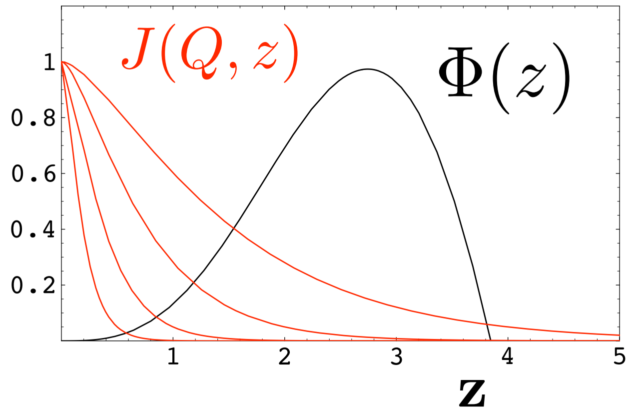
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

**Dimensional Quark Counting Rules:
General result from
AdS/CFT**

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$. The twist is equal to the number of partons, $\tau = n$.

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1+2\kappa^2z^2\right)\partial_z - Q^2z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1+\frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

Soft Wall Model

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

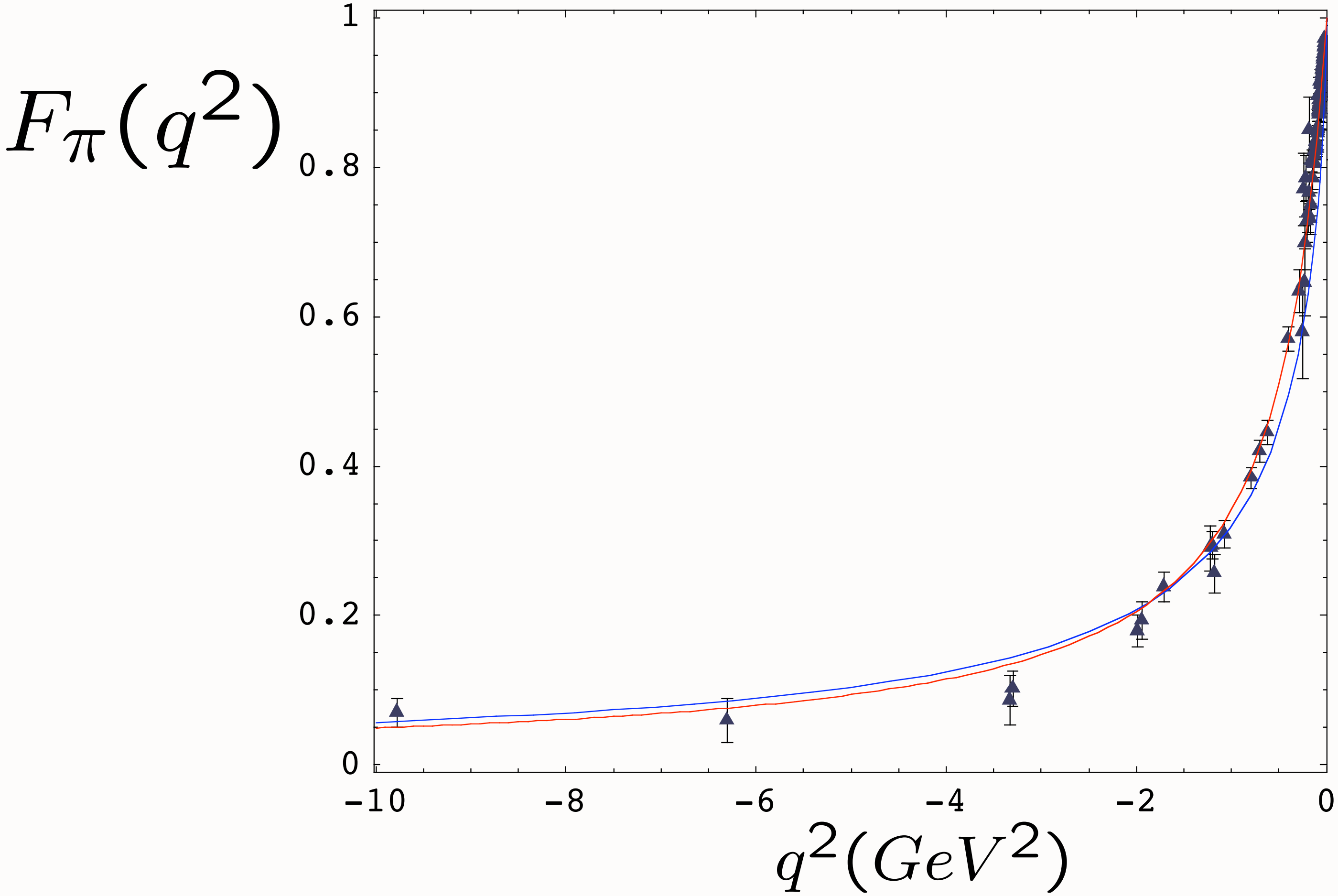
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2}\, \Phi(z) J_\kappa(Q,z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

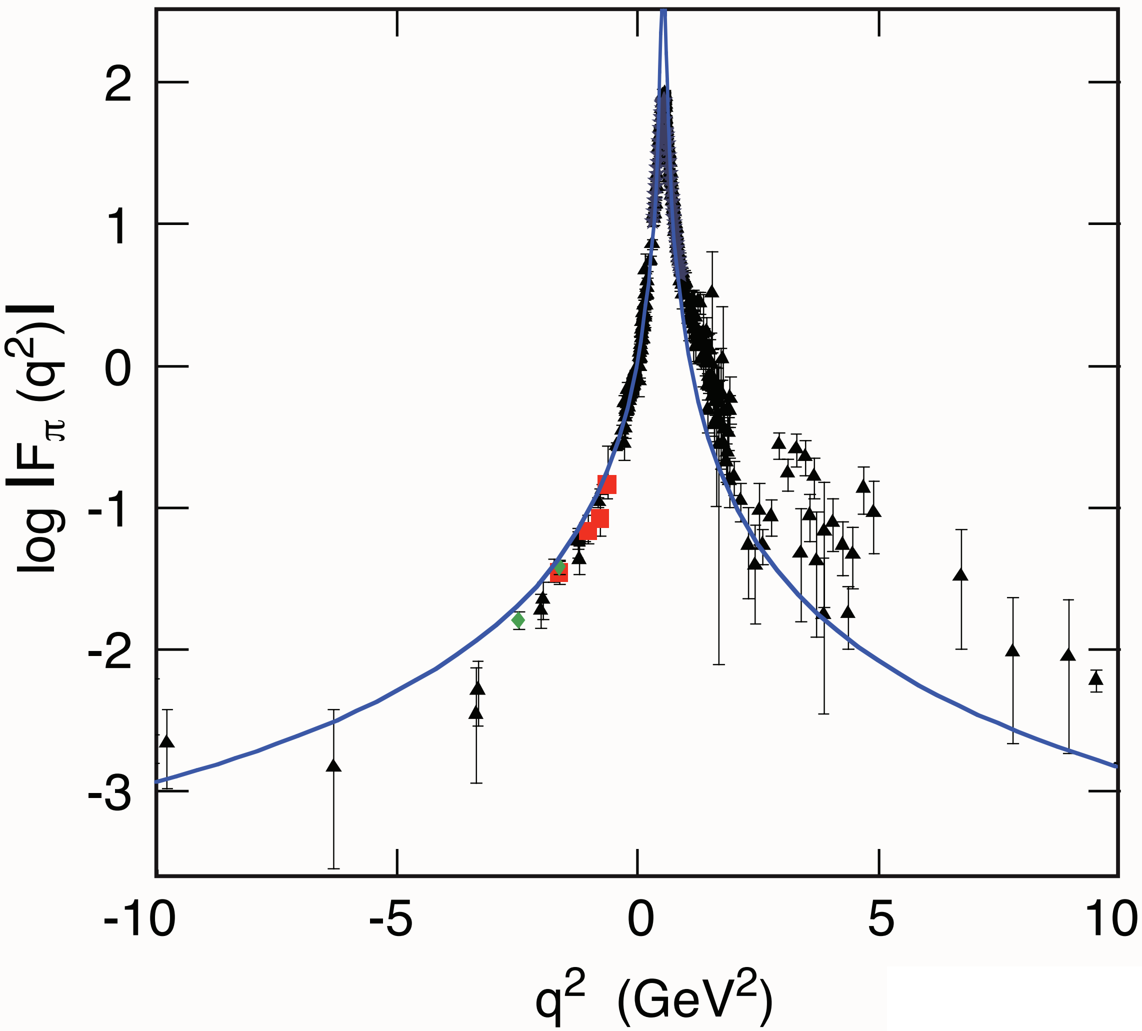
One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

- Analytical continuation to time-like region $q^2 \to -q^2$

$$M_\rho = 2\kappa = 750 \text{ MeV}$$

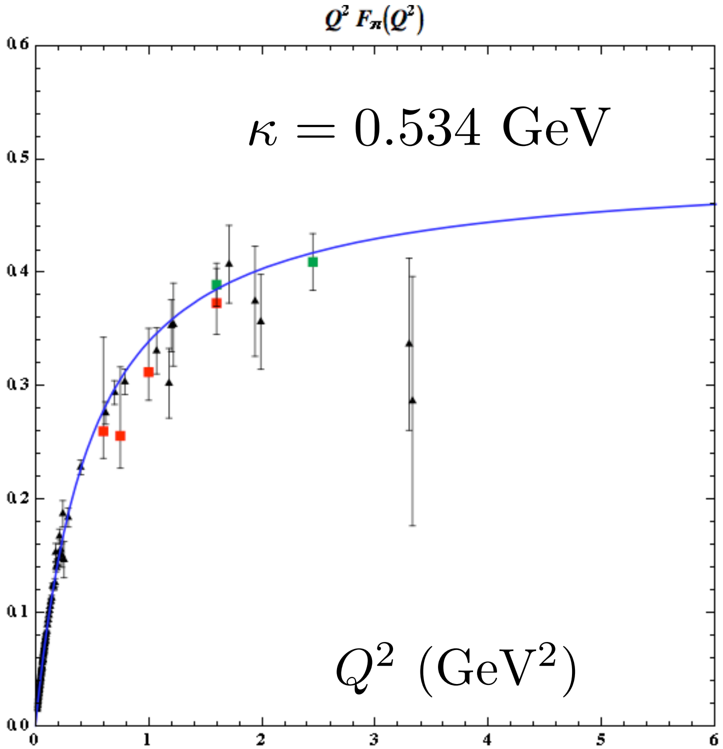
- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).

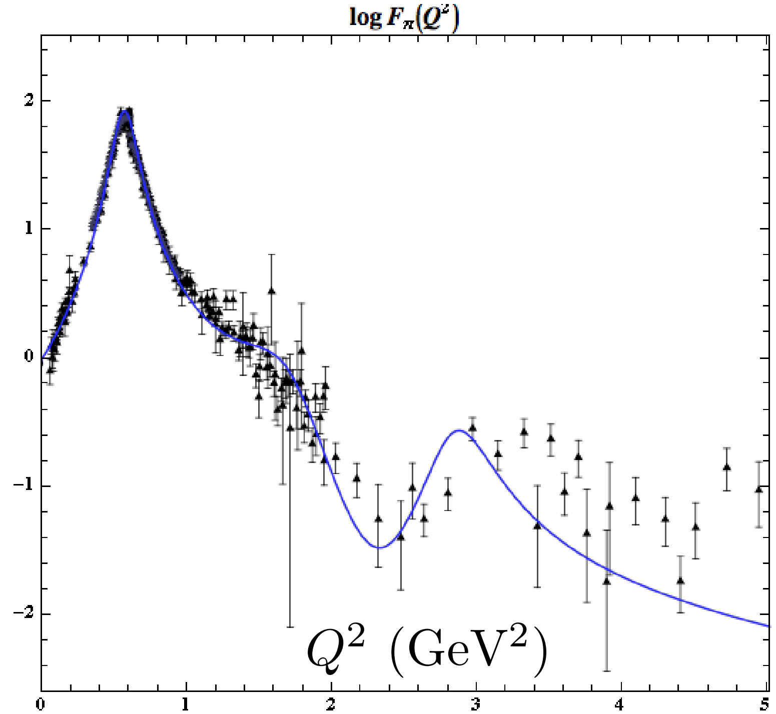


Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

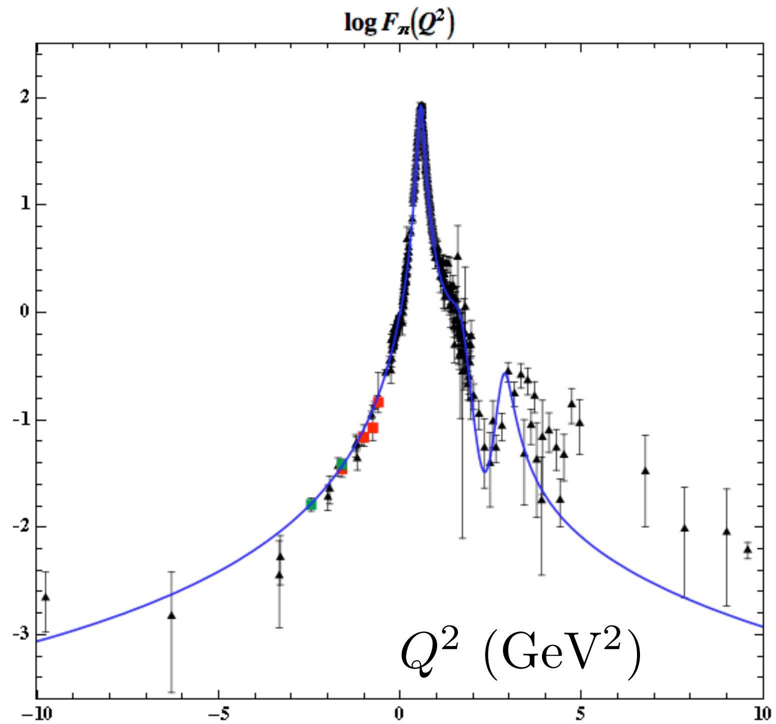
Spacelike and timelike pion form factor





GdT and SJB preliminary

$$|\pi> = \psi_{q\bar q}|q\bar q> + \psi_{q\bar q q\bar q}|q\bar q q\bar q>$$



$$\Gamma_\rho = 120 \text{ GeV}, \Gamma'_\rho = 300 \text{ GeV}$$

$$P_{q\bar q q\bar q} = 15\%$$

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$
\begin{aligned}
\mathcal{M}^2 &= \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions} \\
&= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}
\end{aligned}
$$

Change variables $(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$
\begin{aligned}
\mathcal{M}^2 &= \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} \\
&\quad + \int d\zeta\, \phi^*(\zeta) U(\zeta) \phi(\zeta) \\
&= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)
\end{aligned}
$$

- Functional relation: $\frac{|\phi|^2}{\zeta} = \frac{2\pi}{x(1-x)} |\psi(x, \mathbf{b}_\perp)|^2$
- Invariant mass $\mathcal{M}^2$ in terms of LF mode ϕ

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta\, \phi^*(\zeta) \left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} \right) \phi(\zeta) + \int d\zeta\, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

where the interaction terms are summed up in the effective potential $U(\zeta)$ and the orbital angular momentum in ∇^2 has the $SO(2)$ Casimir representation $SO(N) \sim S^{N-1} :\ L(L+N-2)$

$$-\frac{\partial^2}{\partial \varphi^2} |\phi\rangle = L^2 |\phi\rangle$$

Analogous to nonrelativistic

$$\frac{\ell(\ell+1)}{r^2}$$

- LF eigenvalue equation $H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is a LF wave equation for ϕ

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)}$$

- Effective light-front Schrödinger equation: relativistic, covariant and analytically tractable.

Consider the AdS_5 metric:

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

ds^2 invariant if $x^\mu \to \lambda x^\mu, \; z \to \lambda z,$

Maps scale transformations to scale changes of the the holographic coordinate z.

We define light-front coordinates $x^\pm = x^0 \pm x^3$.

Then $\eta^{\mu\nu} dx_\mu dx_\nu = dx_0{}^2 - dx_3{}^2 - dx_\perp{}^2 = dx^+ dx^- - dx_\perp{}^2$

and

$$\boxed{ds^2 = -\frac{R^2}{z^2}(dx_\perp{}^2 + dz^2) \text{ for } x^+ = 0.}$$

Light-Front/AdS$_5$ Duality

- ds^2 is invariant if $dx_\perp{}^2 \to \lambda^2 dx_\perp{}^2$, and $z \to \lambda z$, at equal LF time.
- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate z.
- Holographic connection of AdS_5 to the light-front.
- The effective wave equation in the two-dim transverse LF plane has the Casimir representation L^2 corresponding to the $SO(2)$ rotation group [The Casimir for $SO(N) \sim S^{N-1}$ is $L(L+N-2)$].

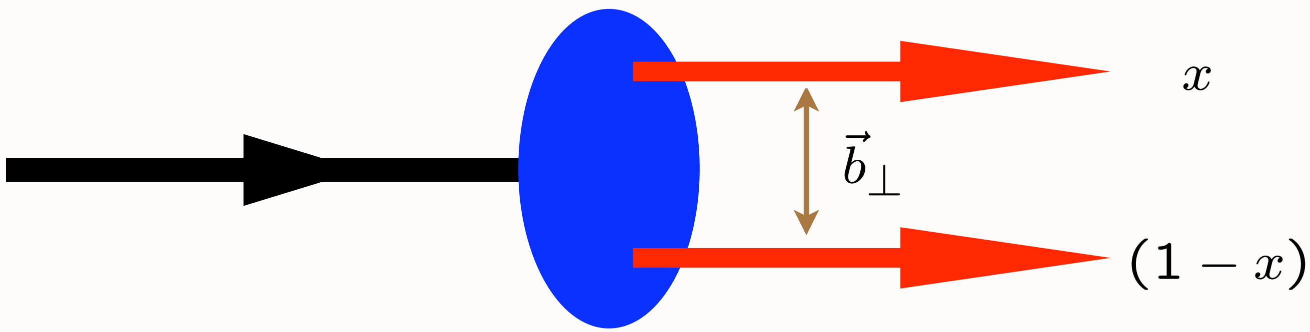
Light-Front Holography:
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

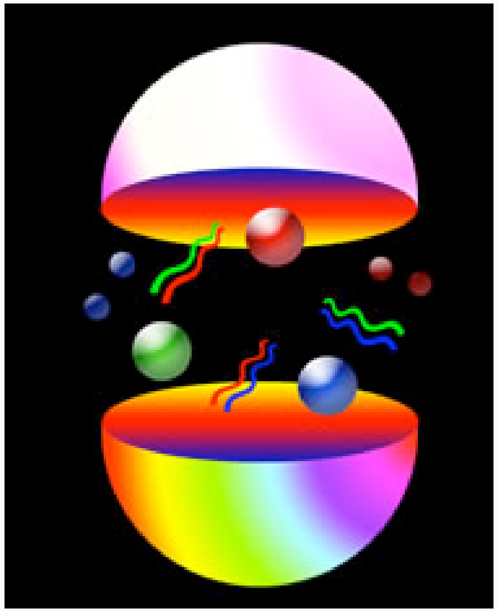
soft wall
confining potential:

- Baryons Spectrum in "bottom-up" holographic QCD

GdT and Sjb hep-th/0409074, hep-th/0501022.

See also T. Sakai and S. Sugimoto

Baryons in Ads/CFT



- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi}, \Psi] = \int d^{d+1}x \, \sqrt{g} \, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z).$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z) = 0$

$$\left[i \left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0.$$

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2}\gamma_5.$$

- Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta}\left[J_{L+1}(\zeta\mathcal{M})u_+ + J_{L+2}(\zeta\mathcal{M})u_-\right].$$

Baryonic modes propagating in AdS space have two components: orbital L and $L+1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm\left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0,$$

given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k}\Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k}\Lambda_{\rm QCD},$$

with a scale independent mass ratio.

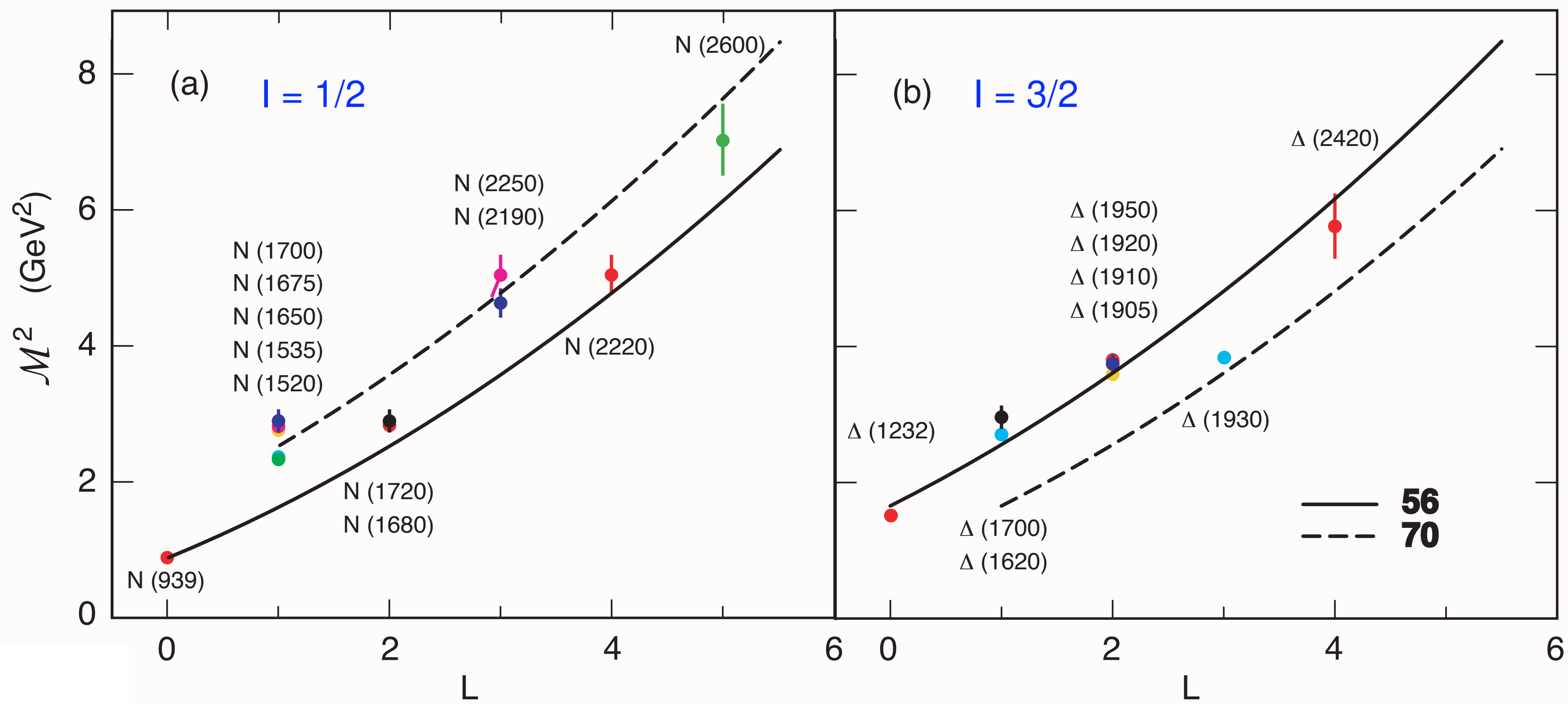


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

SU(6)	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^{+}(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^{+}(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^{-}(1535)\ \ N\frac{3}{2}^{-}(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^{-}(1650)\ \ N\frac{3}{2}^{-}(1700)\ \ N\frac{5}{2}^{-}(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^{-}(1620)\ \ \Delta\frac{3}{2}^{-}(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^{+}(1720)\ \ N\frac{5}{2}^{+}(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^{+}(1910)\ \ \Delta\frac{3}{2}^{+}(1920)\ \ \Delta\frac{5}{2}^{+}(1905)\ \ \Delta\frac{7}{2}^{+}(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^{-}\ \ N\frac{7}{2}^{-}$
	$\frac{3}{2}$	3	$N\frac{3}{2}^{-}\ \ N\frac{5}{2}^{-}\ \ N\frac{7}{2}^{-}(2190)\ \ N\frac{9}{2}^{-}(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^{-}(1930)\ \ \Delta\frac{7}{2}^{-}$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^{+}\ \ N\frac{9}{2}^{+}(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^{+}\ \ \Delta\frac{7}{2}^{+}\ \ \Delta\frac{9}{2}^{+}\ \ \Delta\frac{11}{2}^{+}(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^{-}\ \ N\frac{11}{2}^{-}(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^{-}\ \ N\frac{9}{2}^{-}\ \ N\frac{11}{2}^{-}\ \ N\frac{13}{2}^{-}$

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu}e^{-\kappa^2\zeta^2/2}L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu}e^{-\kappa^2\zeta^2/2}L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

- Eigenvalues

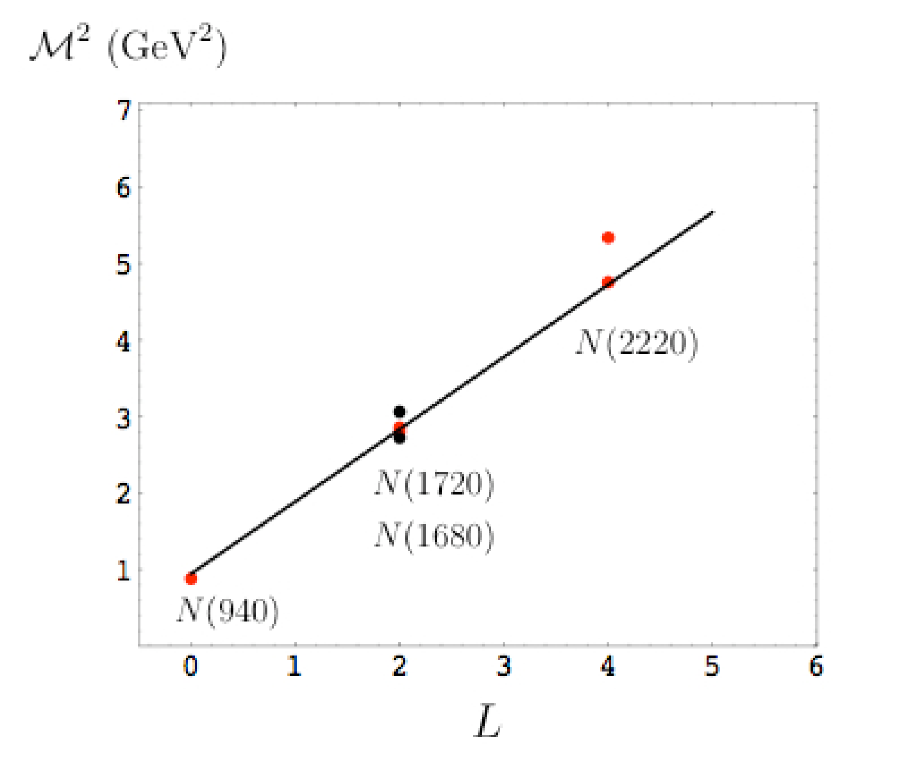
$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

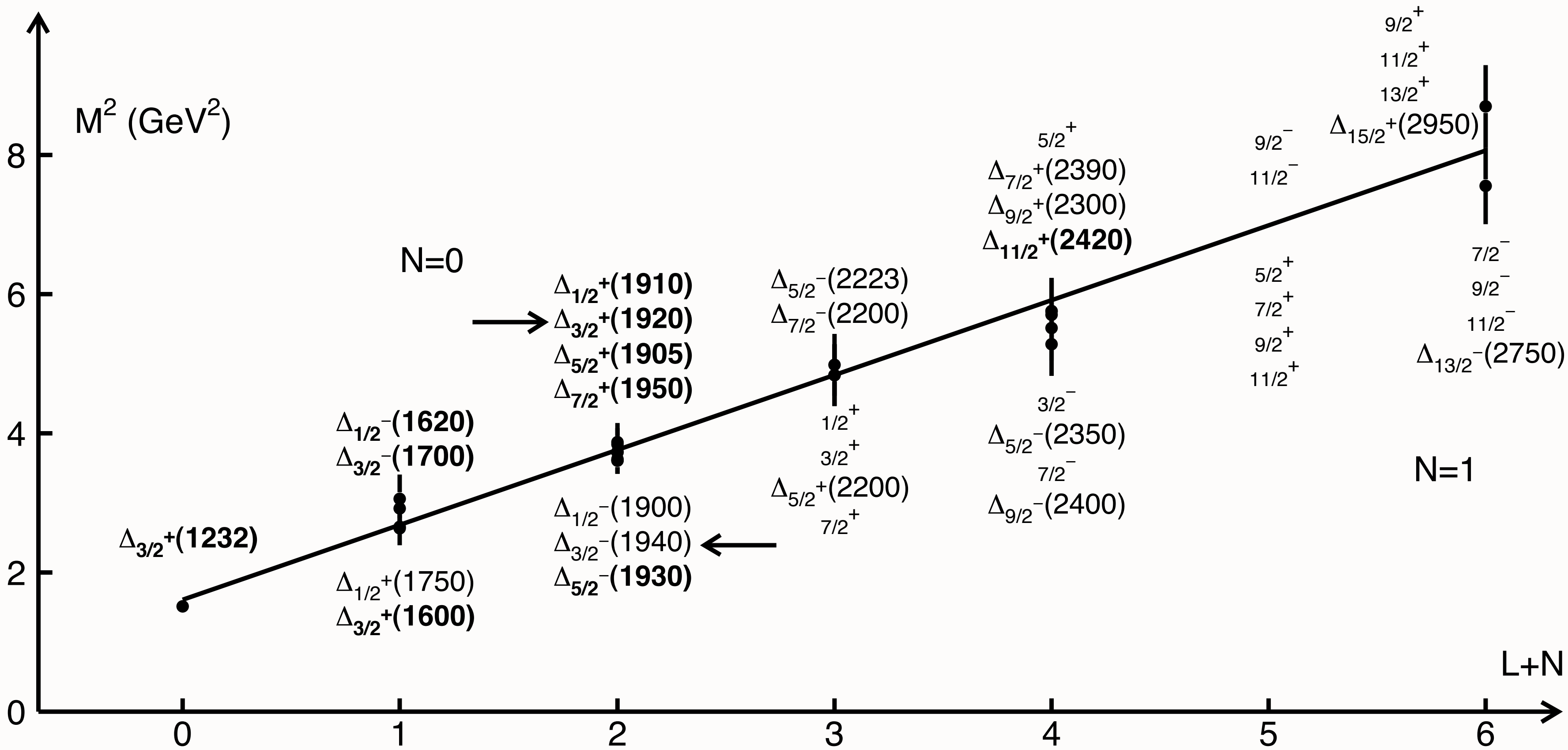
$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Define the zero point energy (identical as in the meson case) $\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2$:

$$\mathcal{M}^2 = 4\kappa^2 (n + L + 1).$$



Proton Regge Trajectory $\quad \kappa = 0.49$GeV



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$
\begin{aligned}
F_+(Q^2) &= g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_-(Q^2) &= g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,
\end{aligned}
$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

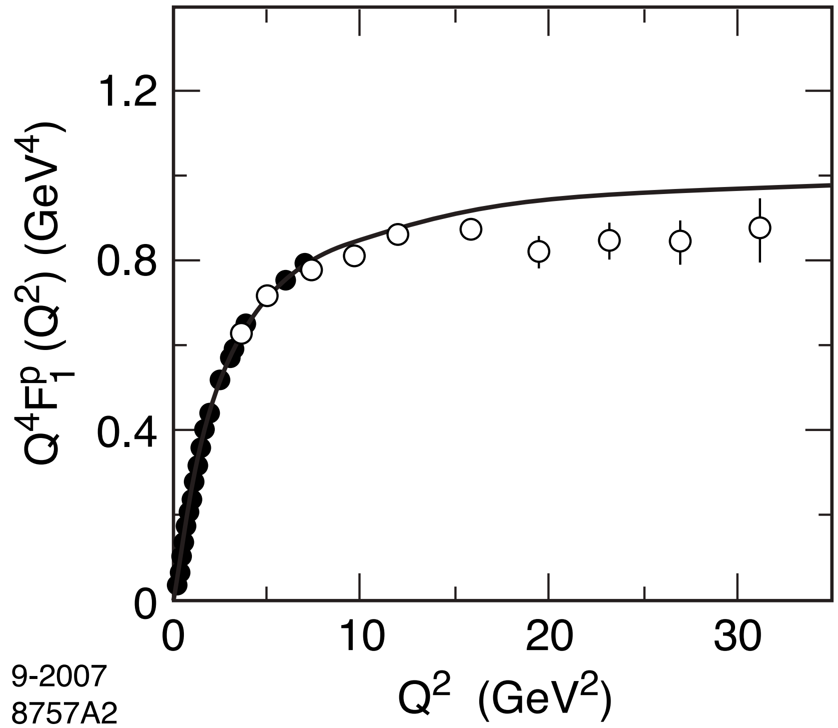
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$
\begin{aligned}
F_1^p(Q^2) &= \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],
\end{aligned}
$$

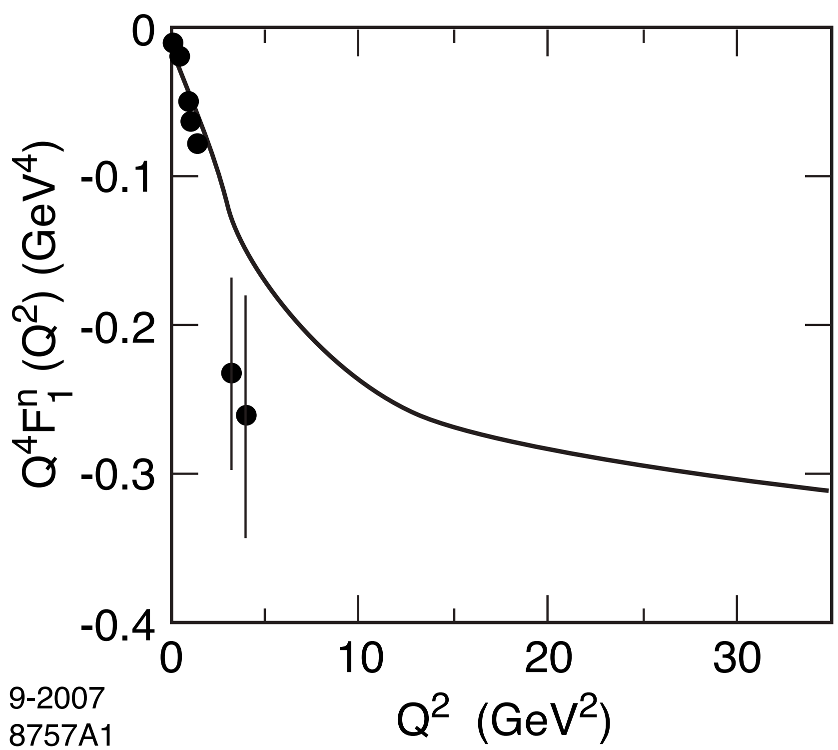
where $F_1^p(0) = 1,\; F_1^n(0) = 0.$

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

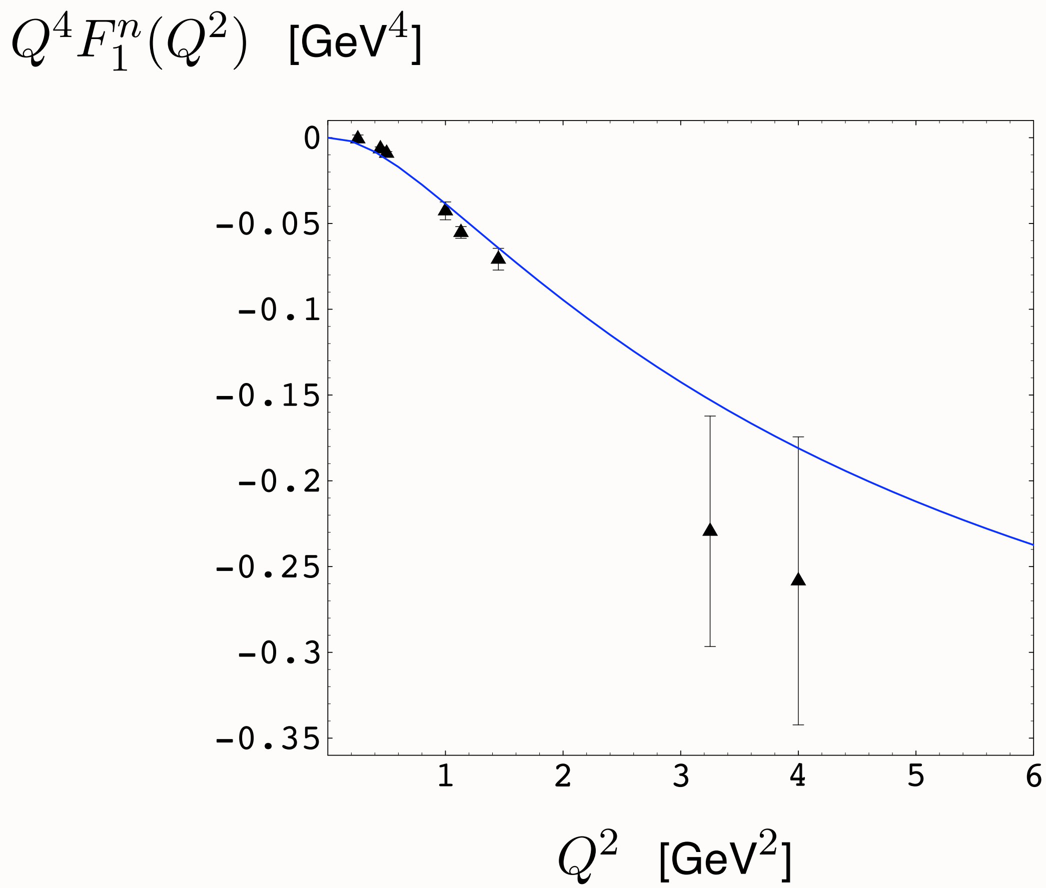


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Dirac Neutron Form Factor

(Valence Approximation)

Truncated Space Confinement

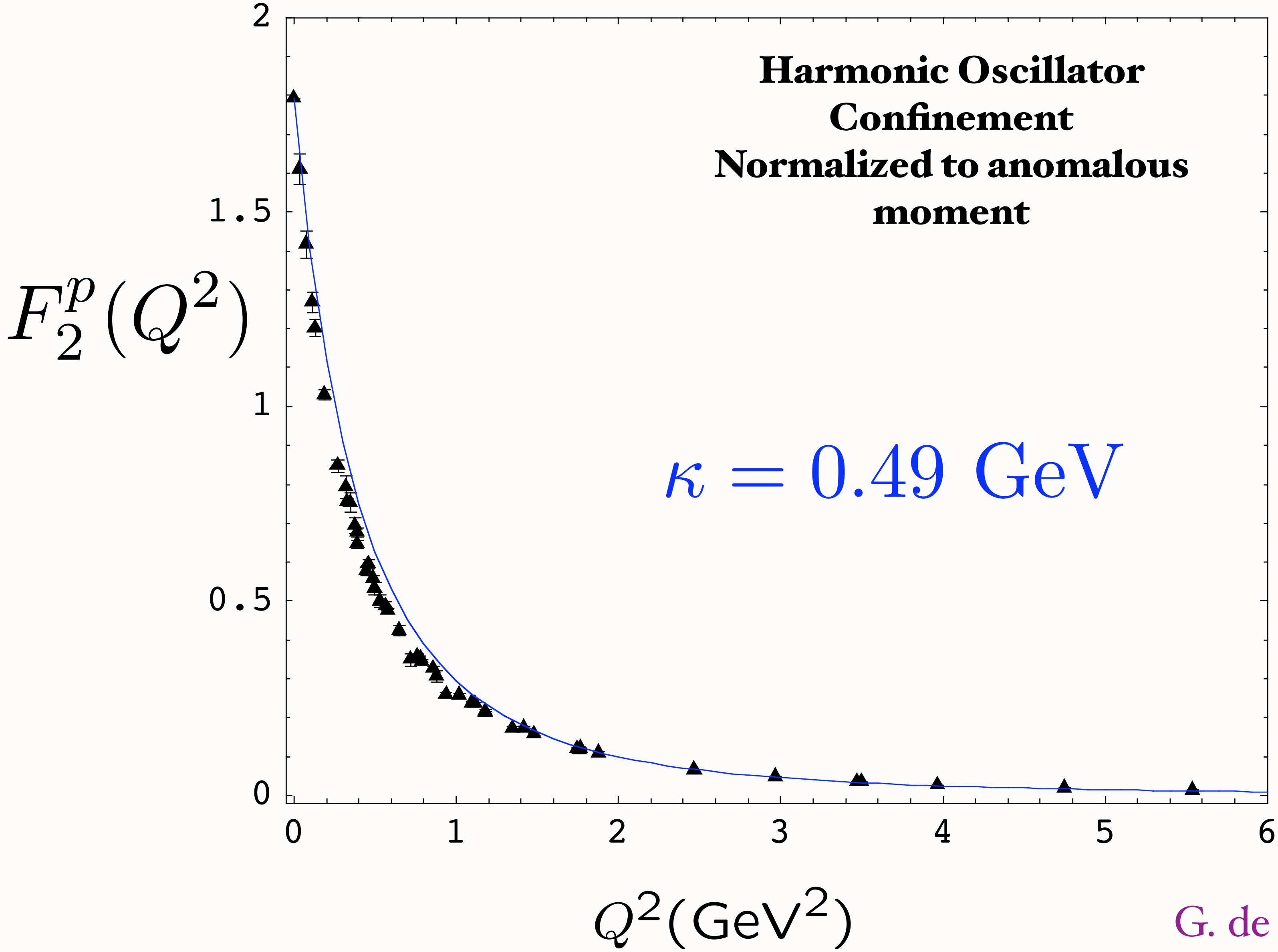


Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



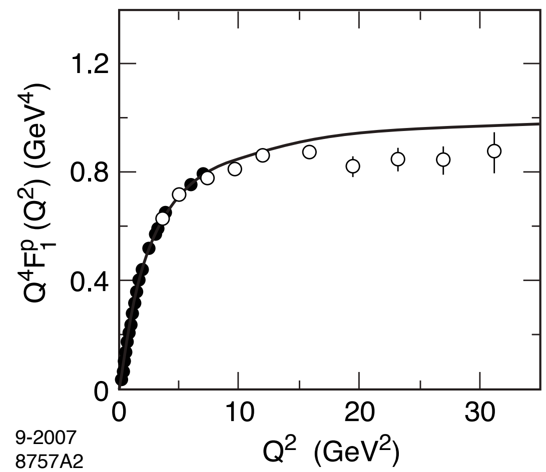
G. de Teramond, sjb

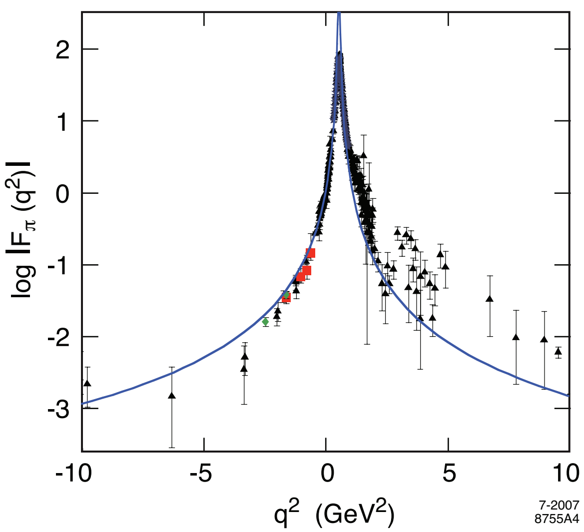
Other Applications of Light-Front Holography

- Nucleon form-factors: space-like region
- Pion form-factors: space and time-like regions
- Gravitational form-factors of composite hadrons

Future Applications

- Pauli Form Factor
- Introduction of massive quarks
- Systematic improvement (QCD Coulomb forces . . .)





SJB and GdT, PLB **582**, 211 (2004)

GdT and SJB, PRL **94**, 201601 (2005)

SJB and GdT, PRL **96**, 201601 (2006)

SJB and GdT, PRD **77**, 056007 (2008)

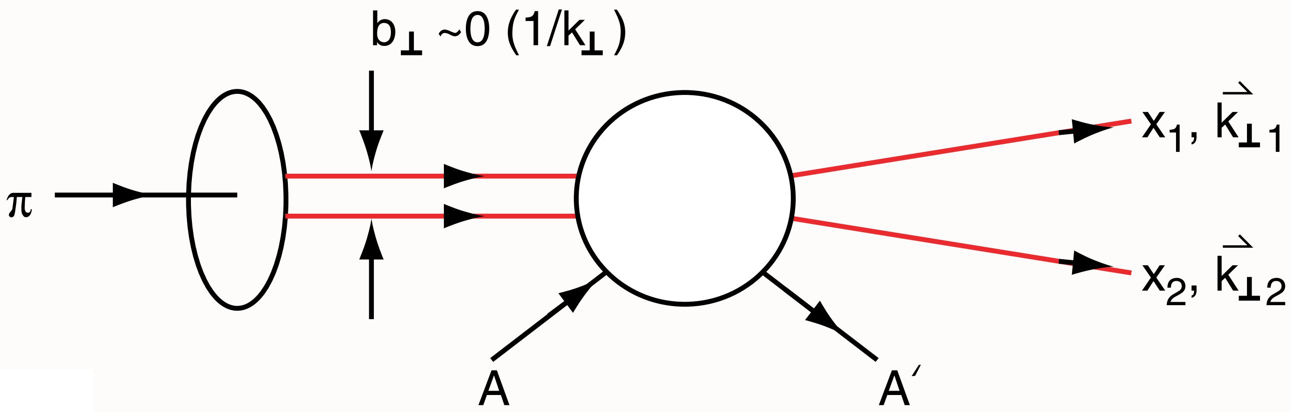
SJB and GdT, PRD **78**, 025032 (2008)

GdT and SJB, PRL **102**, 081601 (2009)

GdT and SJB, arXiv:0809.489

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



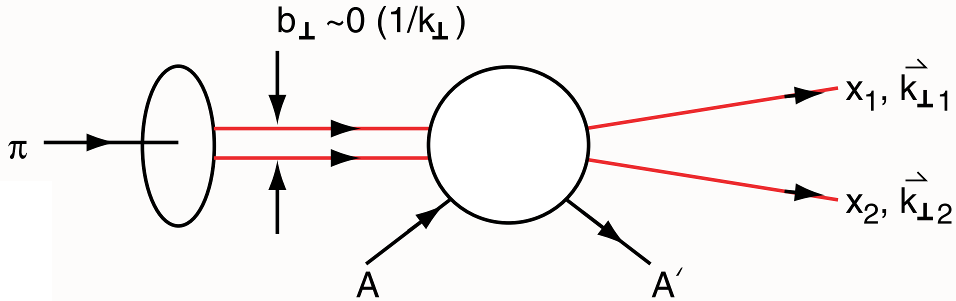
$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

Nucleus left Intact!

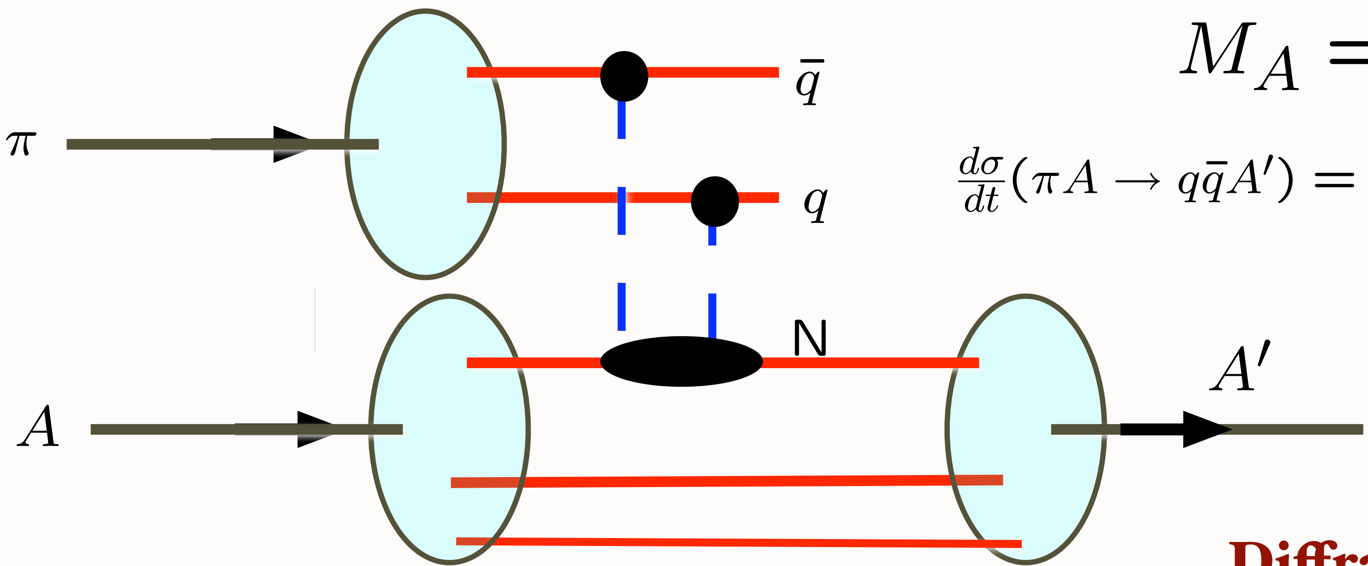
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency



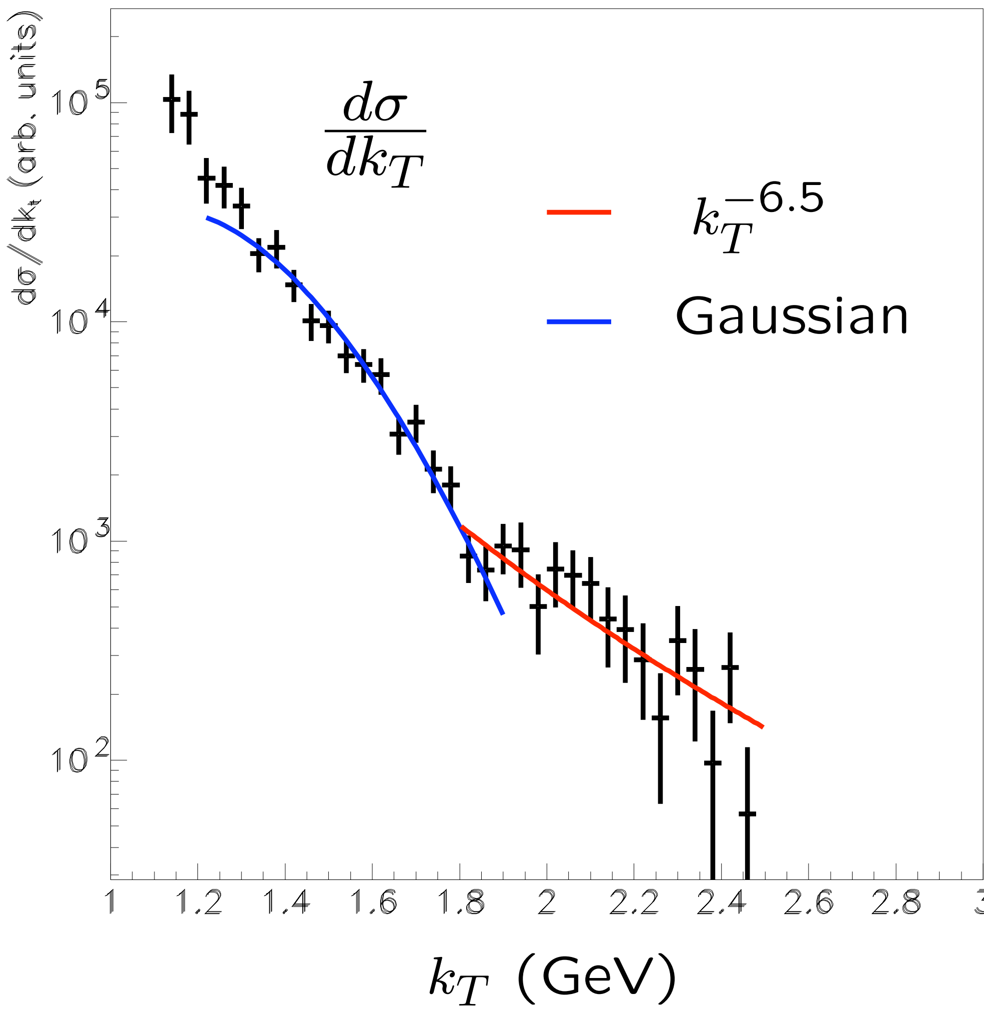
$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F^2_A(t)$$

Target left intact

Diffraction, Rapidity gap

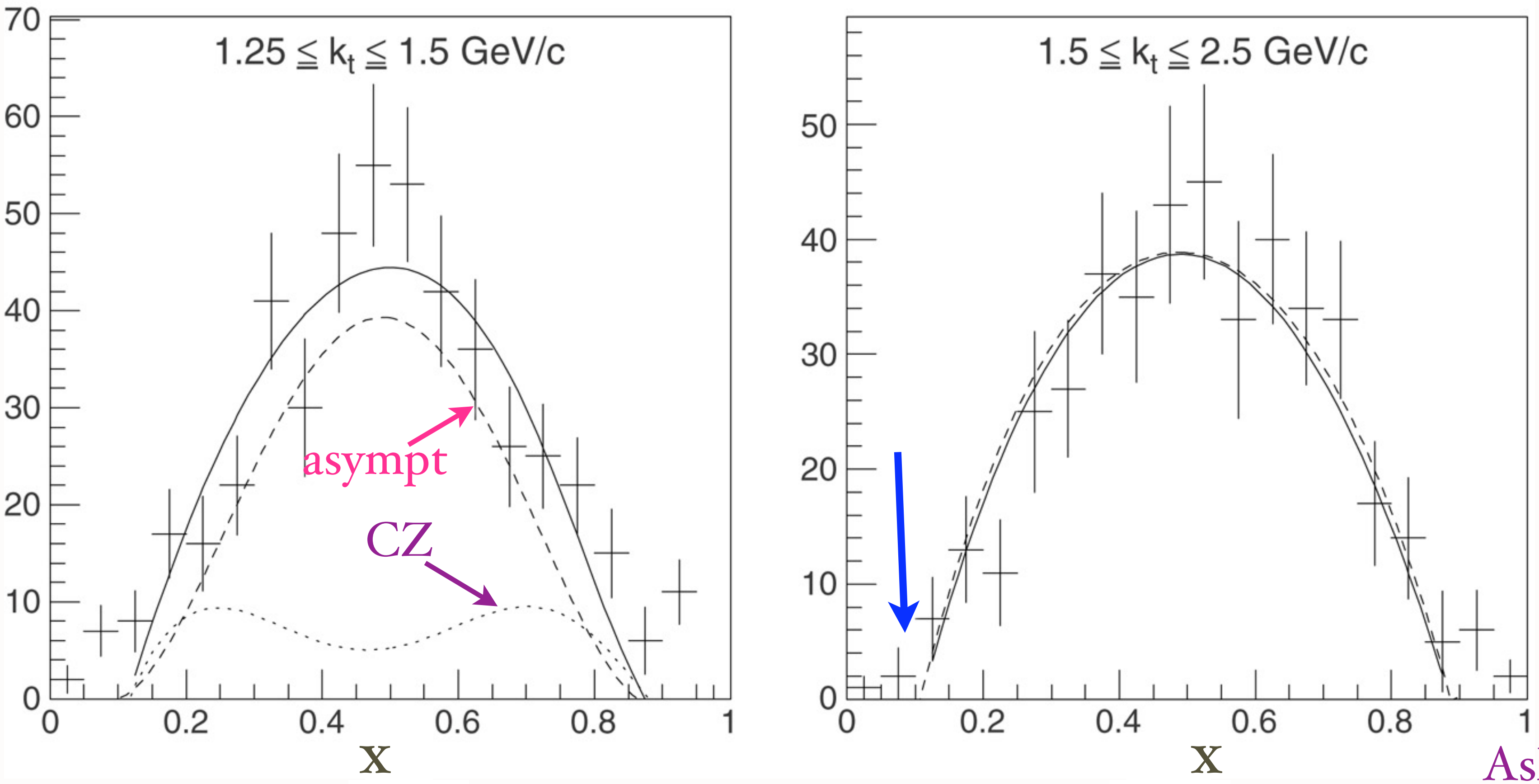
E791 Diffractive Di-Jet transverse momentum distribution



Two Components

High Transverse momentum dependence consistent with PQCD, ERBL Evolution $k_T^{-6.5}$

Gaussian component similar to AdS/CFT HO LFWF



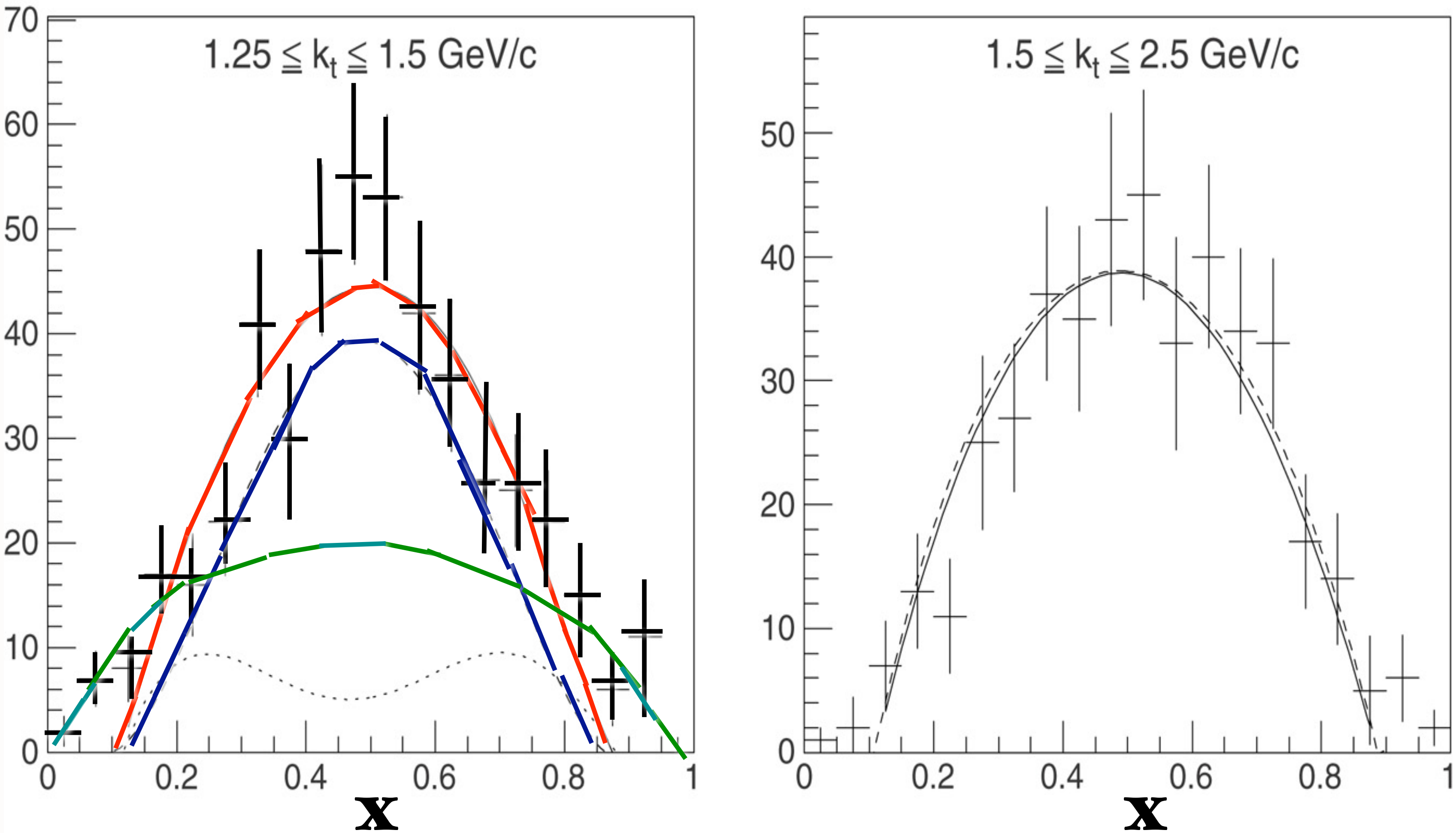
Ashery E791

Narrowing of x distribution at higher jet transverse momentum

x: distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components: Nonperturbative (AdS/CFT) and Perturbative (ERBL) Evolution to asymptotic distribution

$$\phi(x) \propto \sqrt{x(1-x)}$$



Ashery
E791

Possibly two components:

Perturbative (ERBL) + Nonperturbative (AdS/CFT)

$$\phi(x) = A_{\rm pert}(k^2_\perp)x(1-x) + B_{\rm nonpert}(k^2_\perp)\sqrt{x(1-x)}$$

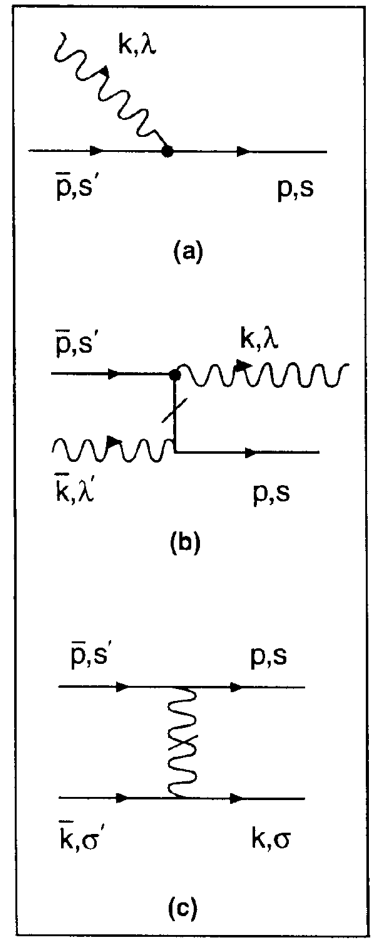
Narrowing of x distribution at high jet transverse momentum

Light-Front QCD

Heisenberg Matrix Formulation

Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$



$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

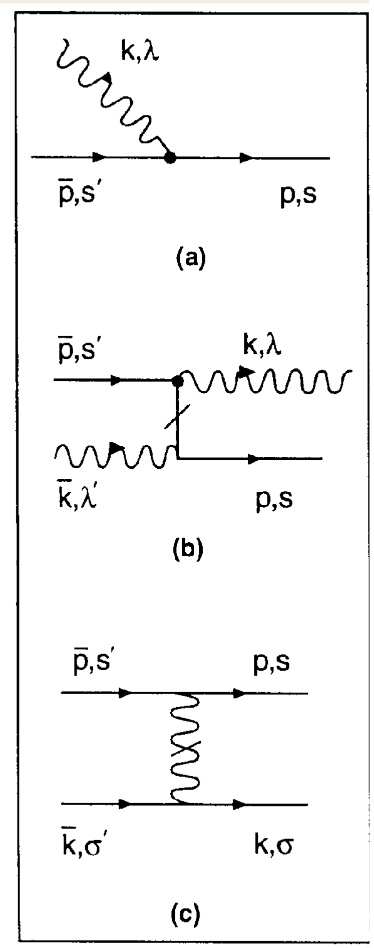
H^{int}_{LF}: Matrix in Fock Space

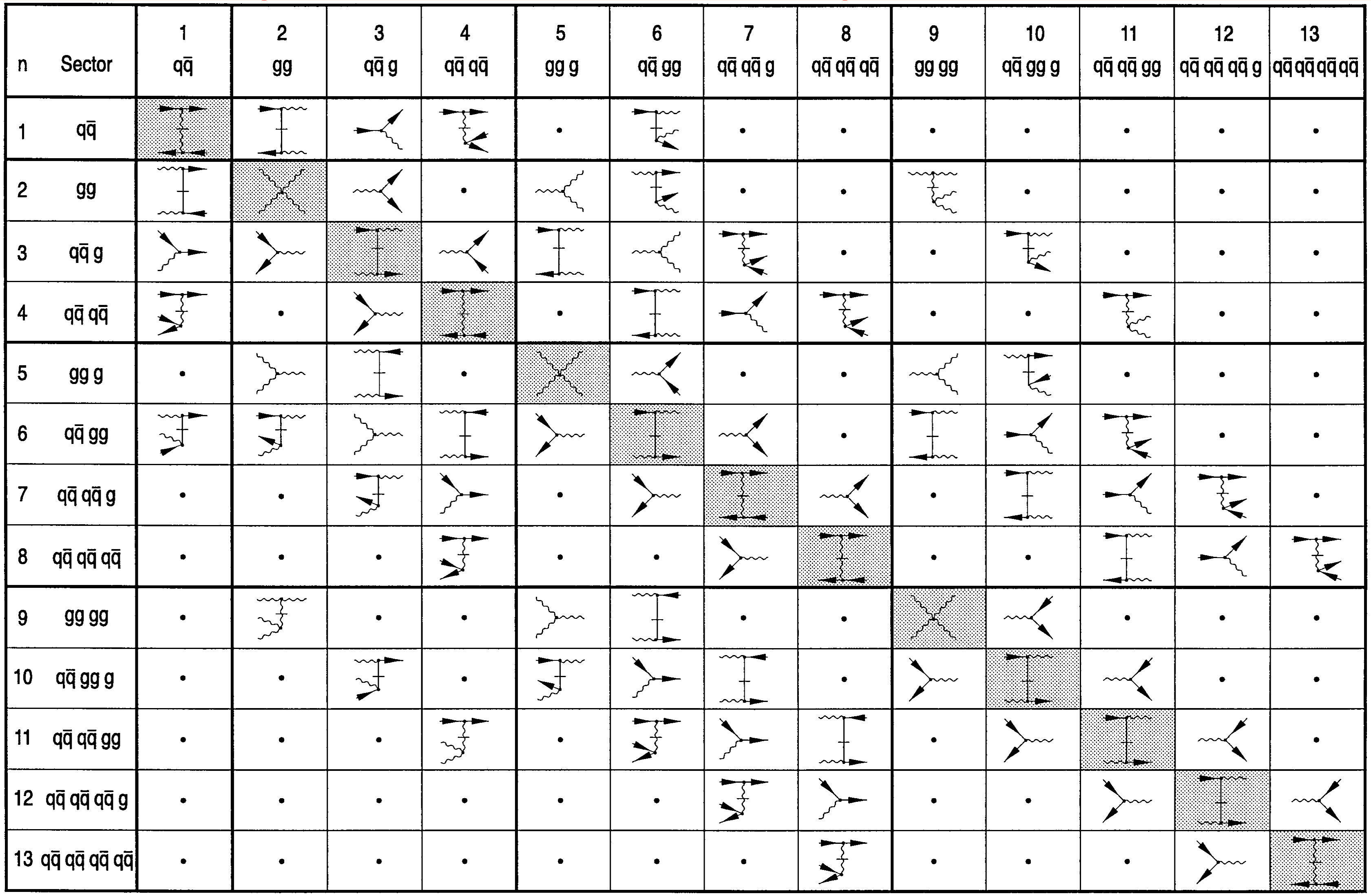
$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Light-Front QCD Heisenberg Equation

$$H^{QCD}_{LC} |\Psi_h\rangle = \mathcal{M}^2_h \, |\Psi_h\rangle$$





Use AdS/QCD basis functions!

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis

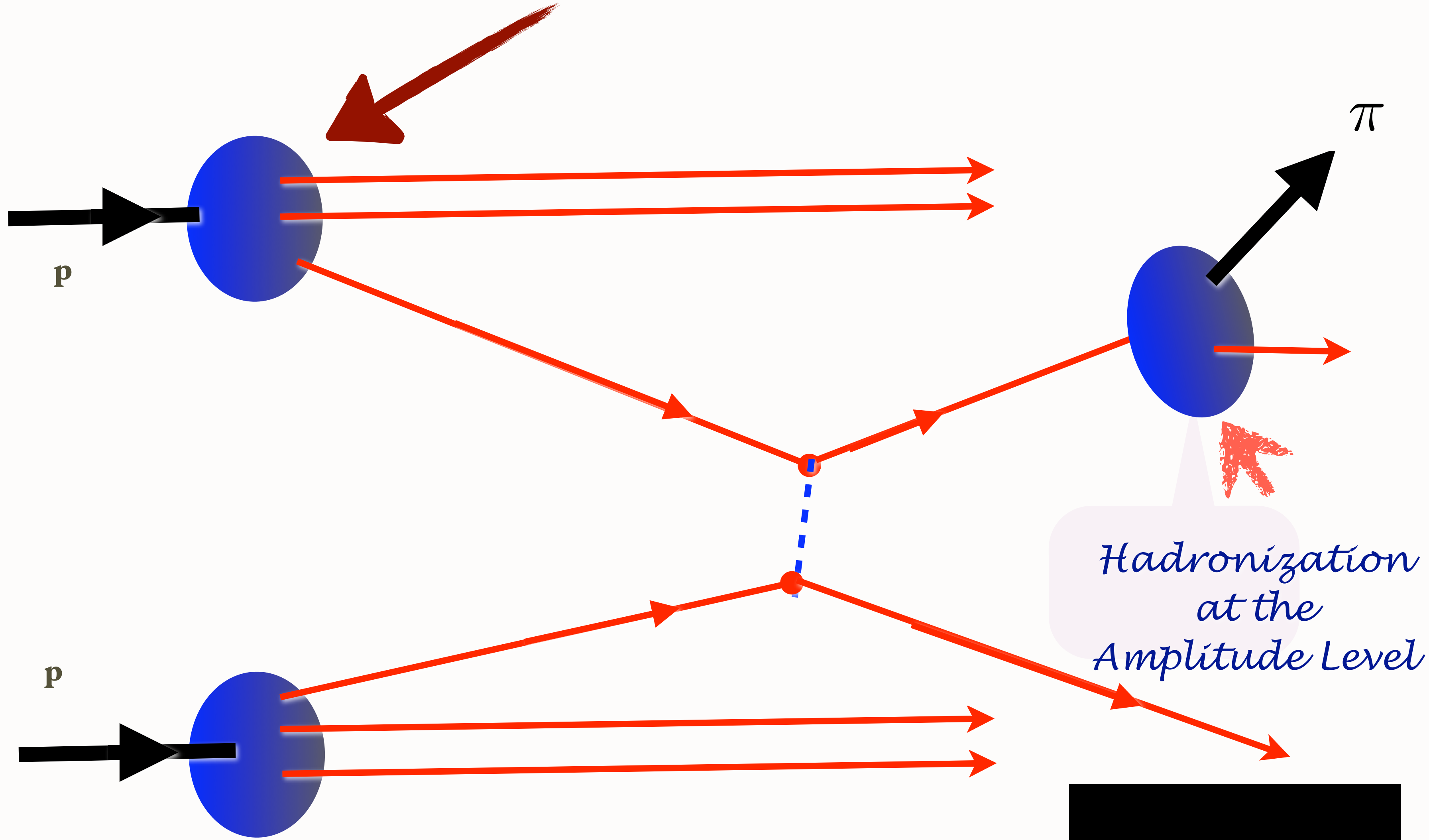
Pauli, Hornbostel, Hiller, McCartor, sjb

- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion

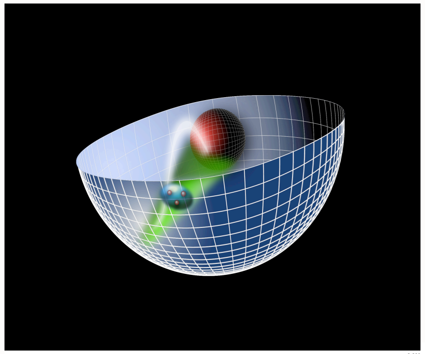
Vary, Harinandrath, Maris, sjb

- Similar to Shell Model calculations

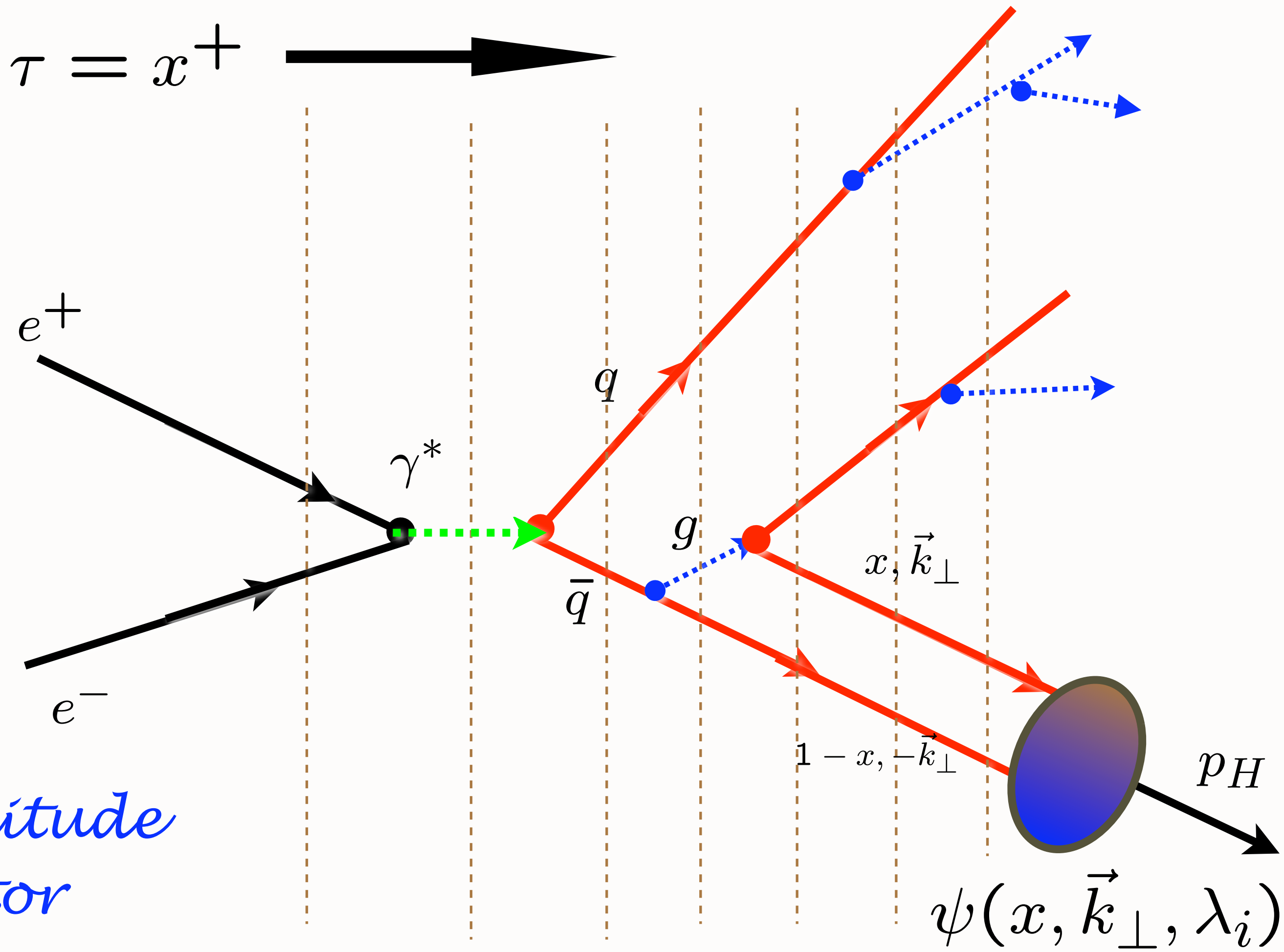
Light-Front Wavefunctions from AdS/CFT



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



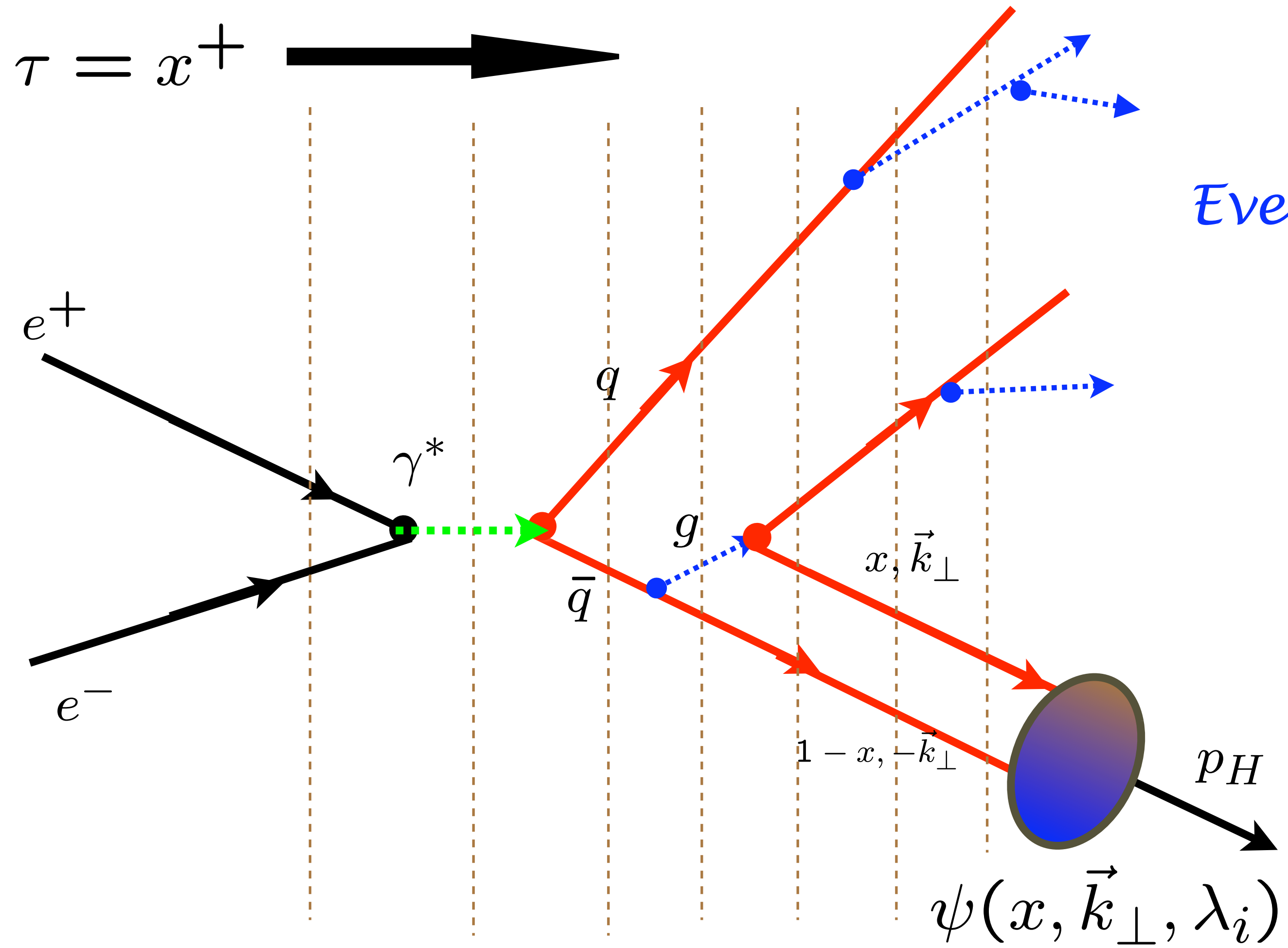
Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



Event amplitude generator

AdS/QCD Hard Wall Confinement:

Capture if $\zeta^2 = x(1-x)b_\perp^2 > \frac{1}{\Lambda^2_{QCD}}$

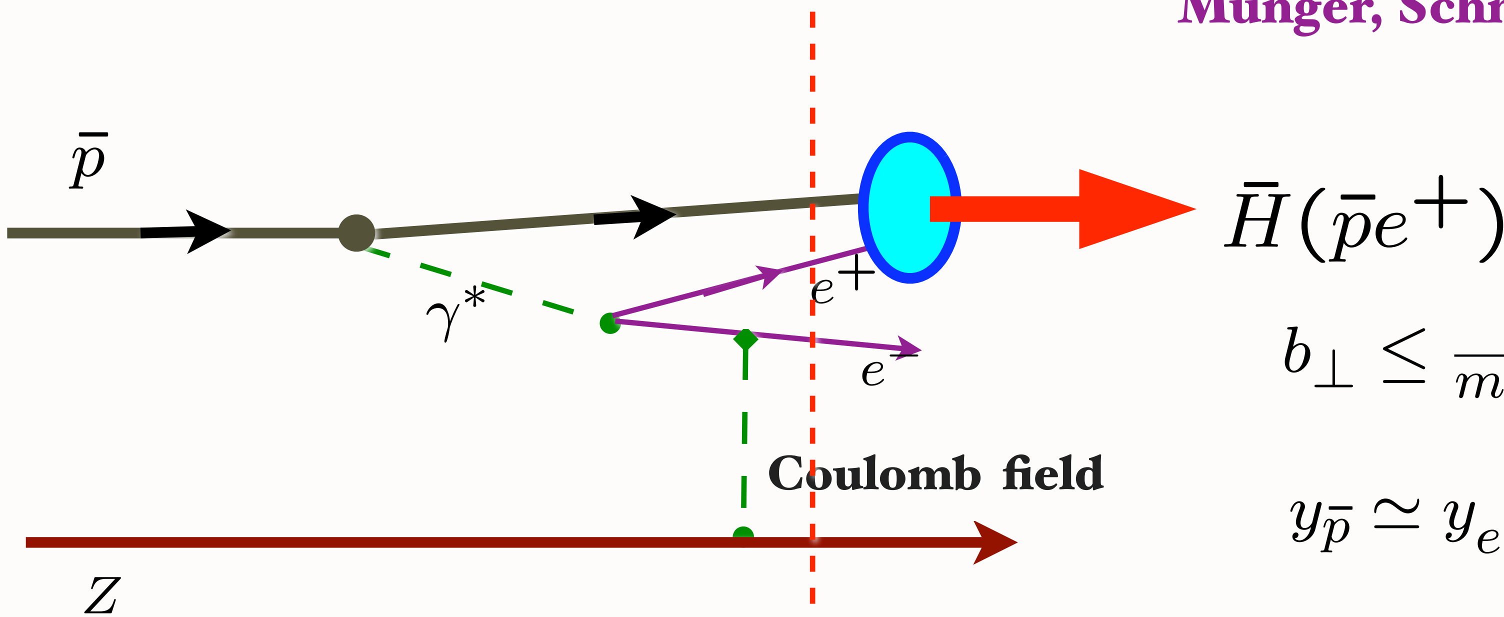
i.e.,

$\mathcal{M}^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda^2_{QCD}$

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

$$y_{\bar{p}} \simeq y_{e^+}$$

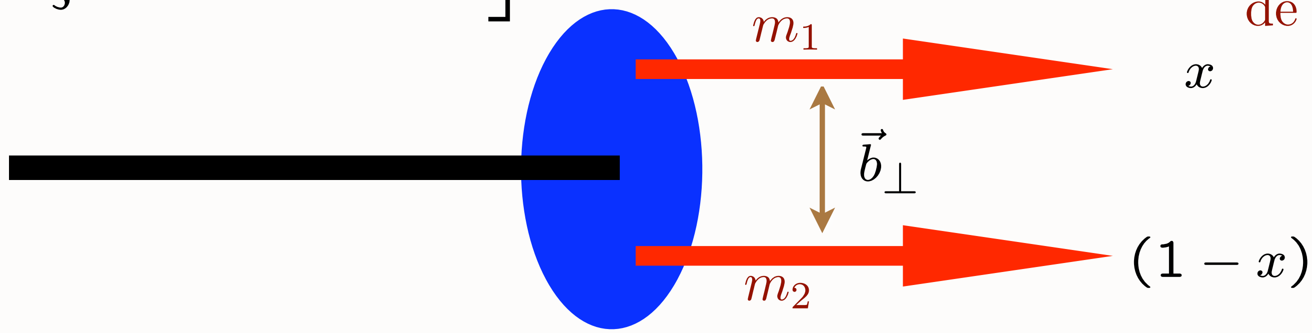
Coalescence of *off-shell* ***co-moving positron and antiproton***

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \rightarrow -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

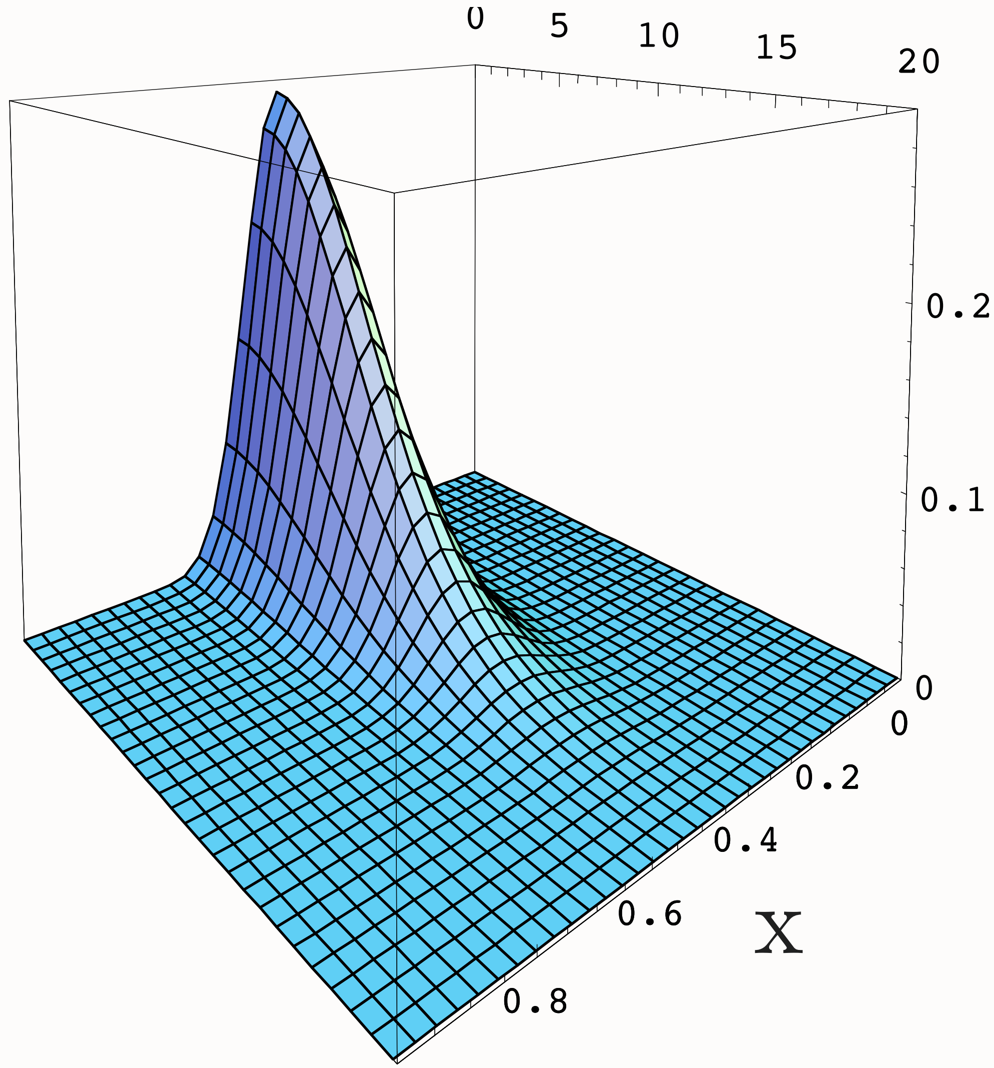
$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}} \sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}]$$

J/ψ

$\psi_{J/\psi}(x,b)$



LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$$\kappa = 0.375 \text{ GeV}$$

$$m_a = m_b = 1.25 \text{ GeV}$$

$|\pi^+ >= |u\bar{d} >$

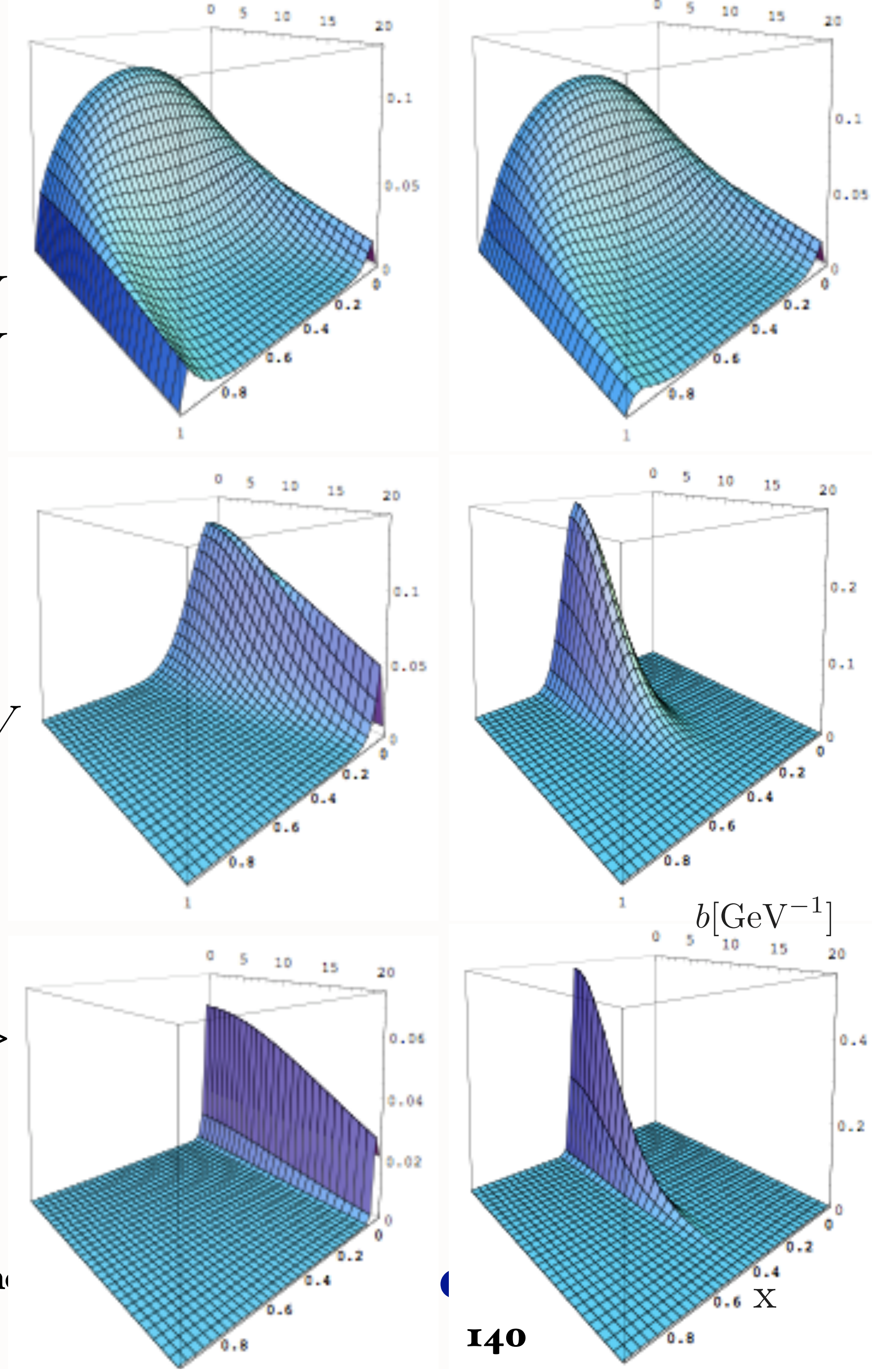
$m_u = 2\ MeV$
$m_d = 5\ MeV$

$|D^+ >= |c\bar{d} >$

$m_c = 1.25\ GeV$

$|B^+ >= |u\bar{b} >$

$m_b = 4.2\ GeV$



$|K^+ >= |u\bar{s} >$

$m_s = 95\ MeV$

$|\eta_c >= |c\bar{c} >$

$|\eta_b >= |b\bar{b} >$

$\kappa = 375\ MeV$

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level

- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances

- Model for LFWFs, meson and baryon spectra: many applications!

- New basis for diagonalizing Light-Front Hamiltonian

- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.

- Quark Interchange dominant force at short distances

Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion (m_q =0)
- Regge Trajectories: universal slope in n and L
- Valid for all integer J & S.
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large Nc limit required
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize H_{LF} on AdS basis

- **DDIS Rescattering: Sivers Effect: Breakdown of Leading-Twist Factorization**
- **Physics of Hard Pomeron**
- **Measure Fundamental Hadron Wavefunction via Di-jet and Tri-jet Fragmentation**
- **Origin of Leading Twist Shadowing**
- **Non-Universal Antishadowing**
- **Heavy quark structure functions at high x**
- **Higgs production at large xF**
- **Hadroproduction of new heavy quark states such as ccu, ccd at high x_F**
- **Novel Nuclear Effects from color structure of IC**
- **Fixed target program at LHC: produce bbb states**
- **Direct Hadroproduction at high pT**

Novel Aspects of QCD

- Anti-Shadowing not universal
- Hadronization at the Amplitude Level
- AdS/QCD Light-Front Wavefunctions
- Fixed-x_T scaling laws
- Multiple renormalization scales
- Initial and Final-State Scattering Effects
- Intrinsic Heavy Quark Fock States
- Direct Higher-Twist Subprocesses and Color Transparency
- Diffractive Reactions at Leading Twist

Novel Aspects of QCD

- Heavy quark distributions do not derive exclusively from DGLAP or gluon splitting -- component intrinsic to hadron wavefunction: Higgs at high x_F

- Initial and final-state interactions are not power suppressed in hard QCD reactions

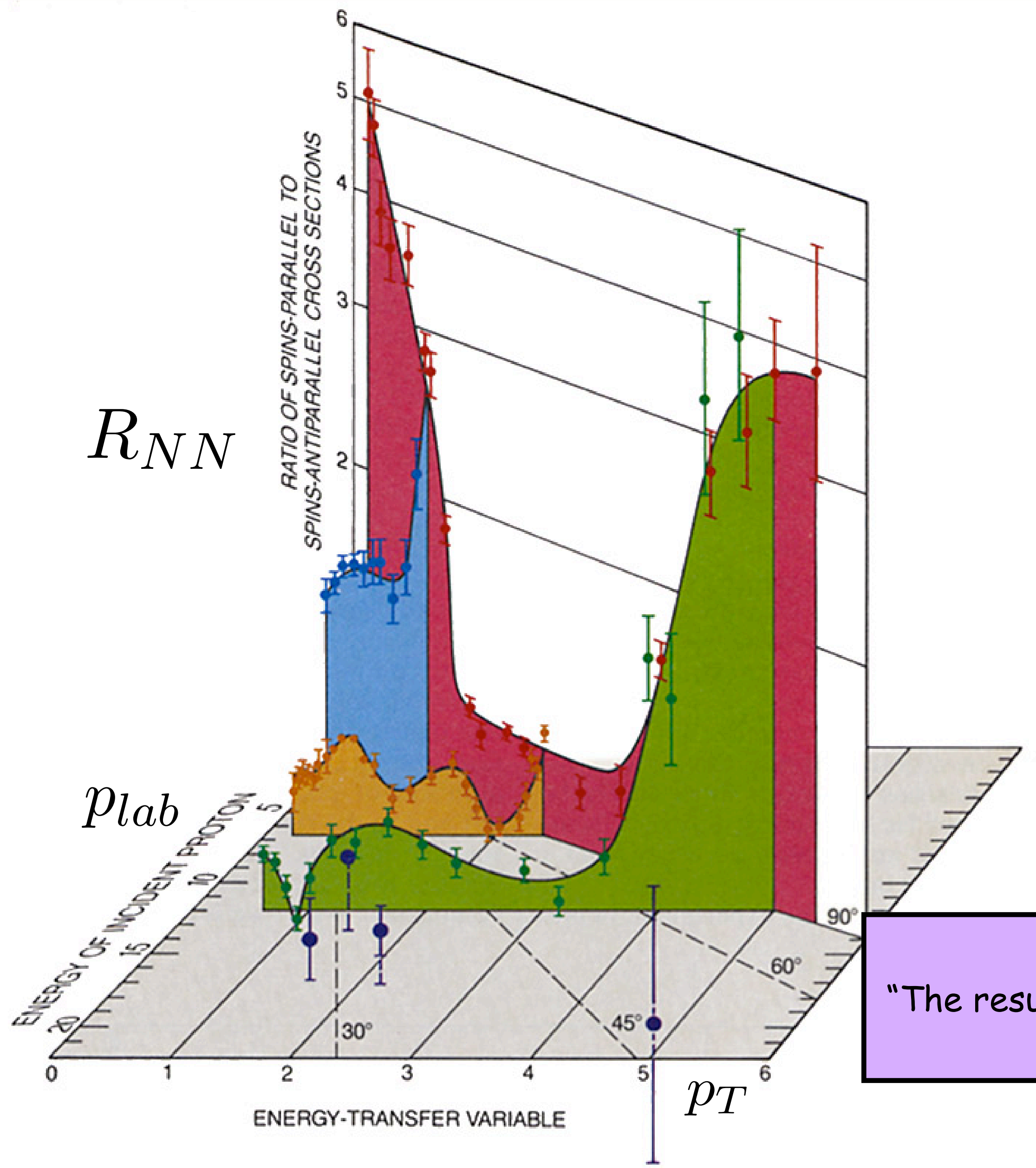
- LFWFS are universal, but measured nuclear parton distributions are not universal -- antishadowing is flavor dependent

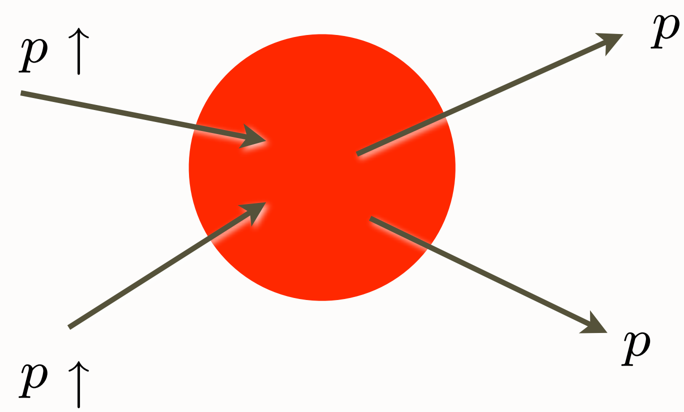
- Hadroproduction at large transverse momentum does not derive exclusively from 2 to 2 scattering subprocesses

The remarkable anomalies of proton-proton scattering

- Double spin correlations
- Single spin correlations
- Color transparency

Spin Correlations in Elastic $p-p$ Scattering





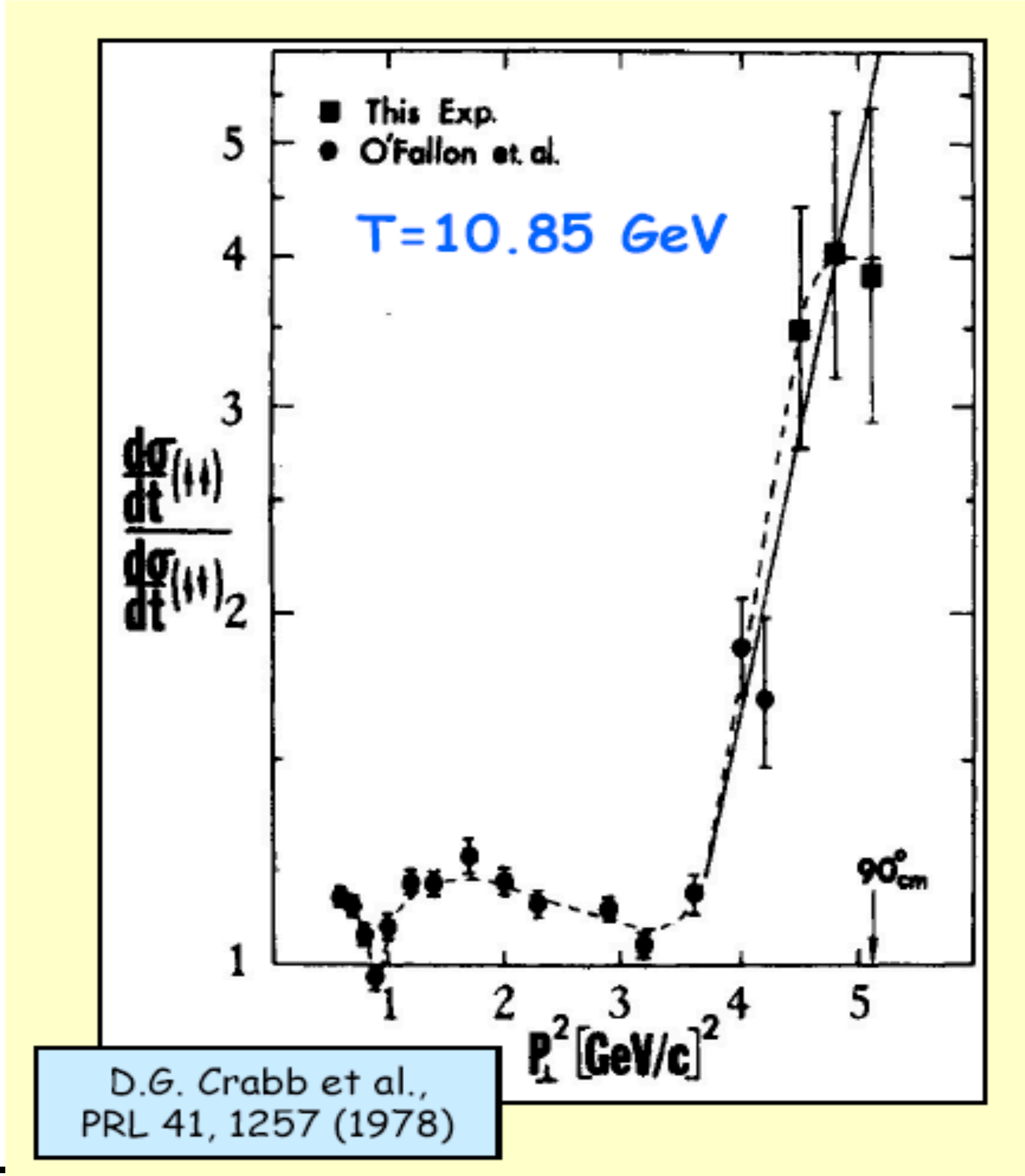
polarization normal to scattering plane

Ratio reaches 4:1 !

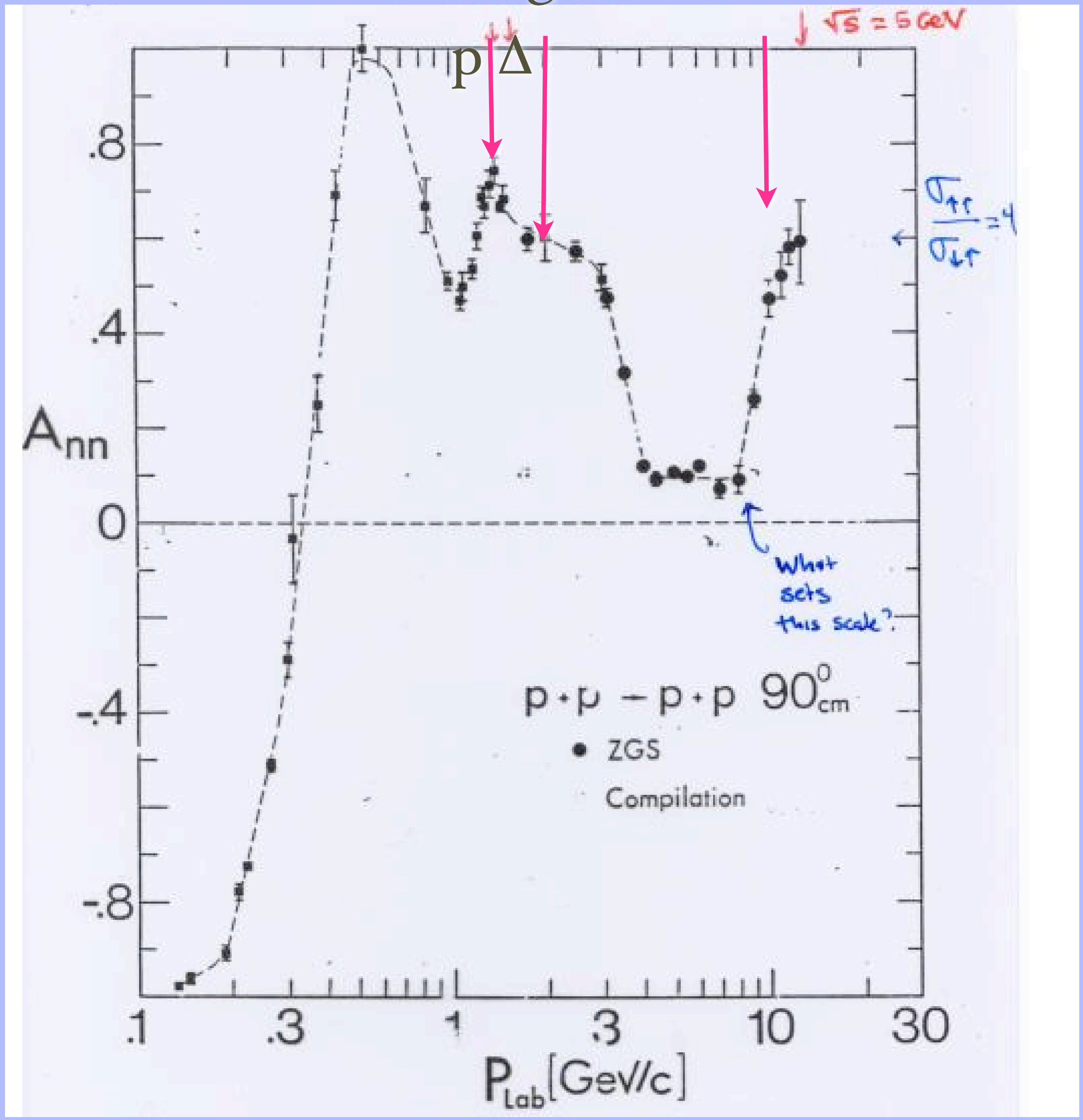
A. Krisch, Sci. Am. 257 (1987)
"The results challenge the prevailing theory that describes the proton's structure and forces"

Unexpected spin effects in pp elastic scattering

larger t region can be explored in $p\bar{p}$



Strangeness Charm



"Exclusive Transversity"

A. Krisch, Sci. Am. 257 (1987)
"The results challenge the prevailing theory that describes the proton's structure and forces"

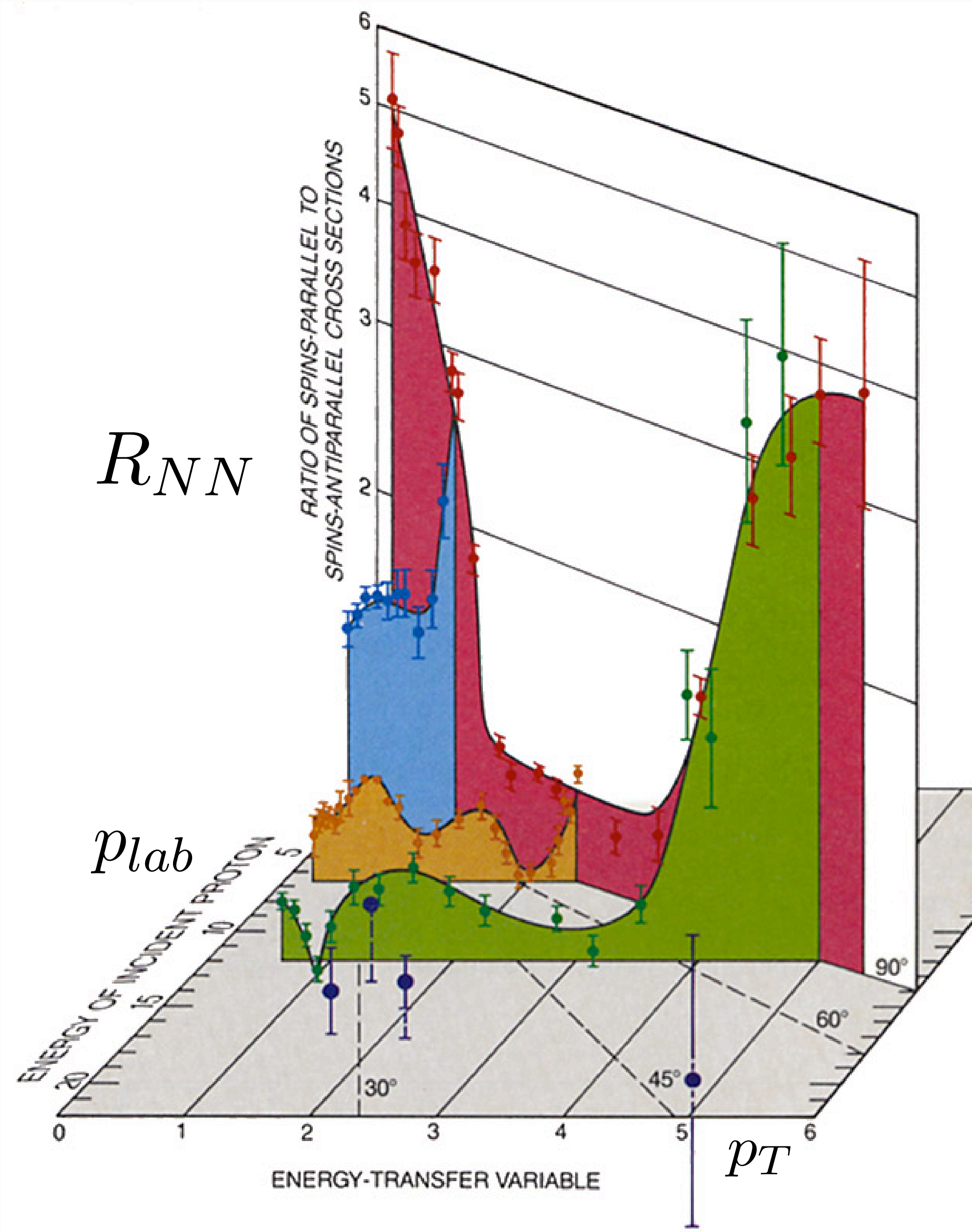
Spin-dependence at large-P_T (90°_{cm}):

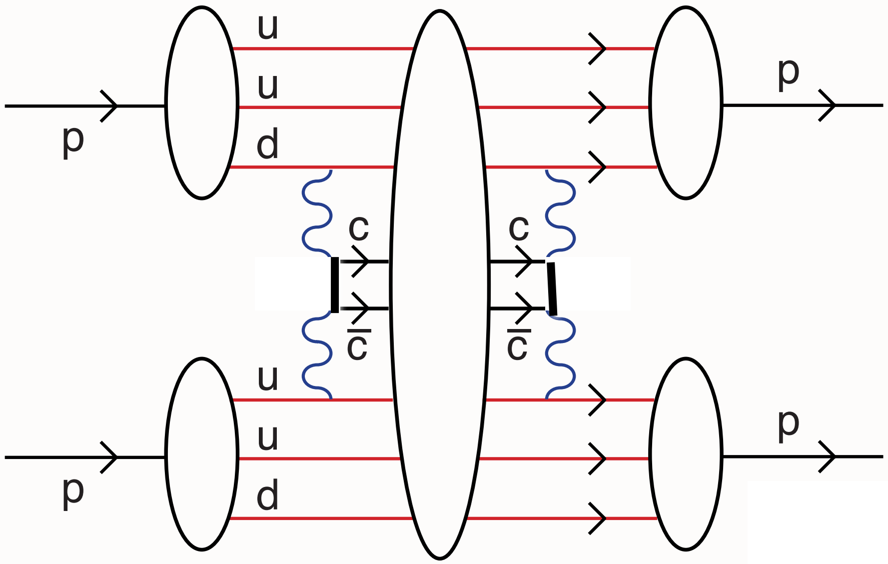
Hard scattering takes place only with spins ↑↑

Coincidence?: Quenching of Color Transparency

Coincidence?: Charm and Strangeness Thresholds

Alternative: Six-Quark Hidden-Color Resonances





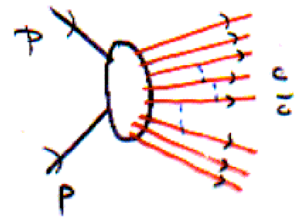
QCD Schwinger-Sommerfeld Enhancement at Heavy Quark Threshold

Hebecker, Kuhn, sjb

S. J. Brodsky and G. F. de Teramond, "Spin Correlations, QCD Color Transparency And Heavy Quark Thresholds In Proton Proton Scattering," Phys. Rev. Lett. **60**, 1924 (1988).

Spin, Coherence at heavy quark thresholds

$p\bar{p} \to Q\bar{Q}\, X$



Strong distortion at threshold $\vec{p}_{rel} \sim 0$

$\sqrt{s}_{Th} = 3+2 \simeq 5$ GeV $\quad pp \to c\bar{c}\, X$

8 quarks in S-wave odd parity!

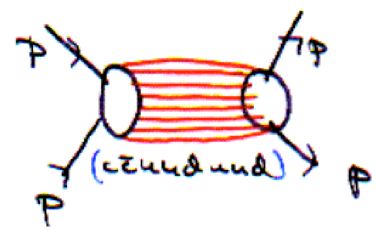
$\therefore\ J = L = S = 1$ for pp

$B = 2$

resonance near threshold?

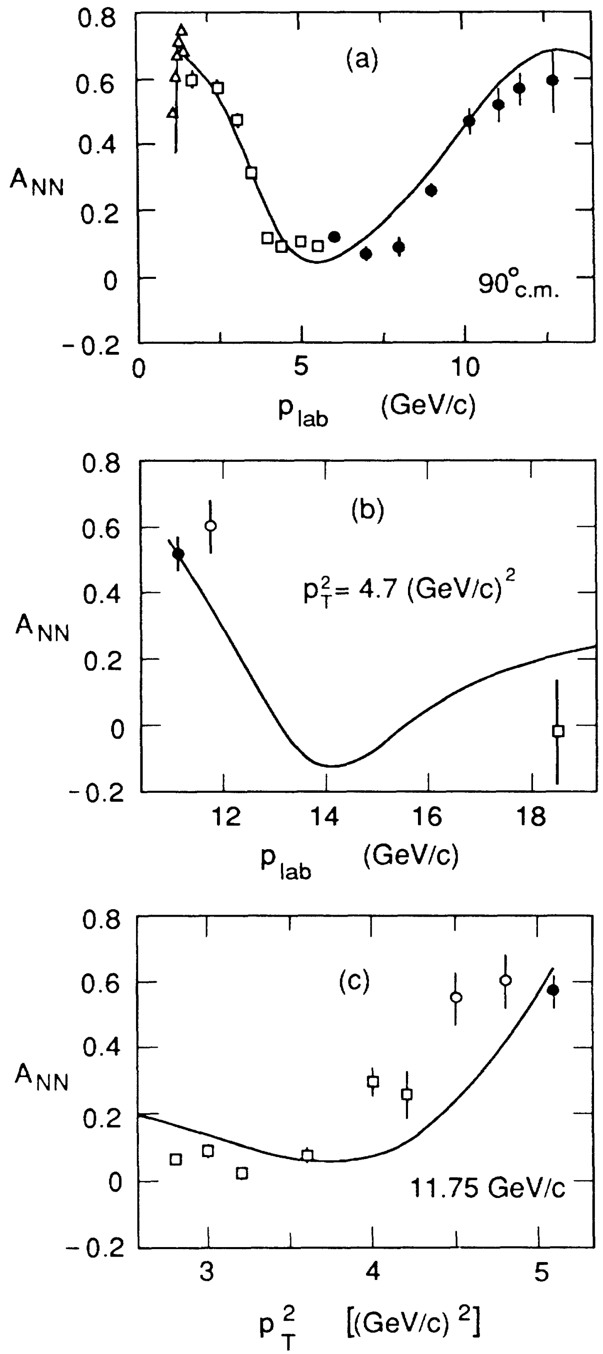
$\frac{d\sigma}{dt}(pp \to pp)$

$\sqrt{s} \sim 5$ GeV



$A_{NN} = 1$ for $J=L=S=1$ $\quad p\uparrow p\uparrow$ only

expect increase of A_{NN} at $\sqrt{s} = 3, 5, 12$ GeV
$\theta_{cm} = 90°$



S. J. Brodsky and G. F. de Teramond, "Spin Correlations, QCD Color Transparency And Heavy Quark Thresholds In Proton Proton Scattering," Phys. Rev. Lett. **60**, 1924 (1988).

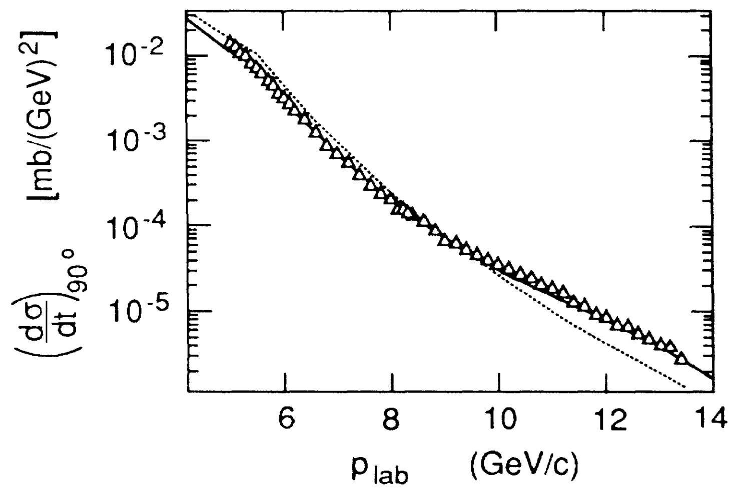
Quark Interchange + 8-Quark Resonance

$|uuduudc\bar{c}>$ Strange and Charm Octoquark!

$M = 3$ GeV, $M = 5$ GeV.

$J = L = S = 1,\ B = 2$

$$A_{NN} = \frac{d\sigma(\uparrow\uparrow) - d\sigma(\uparrow\downarrow)}{d\sigma(\uparrow\uparrow) + d\sigma(\uparrow\downarrow)}$$



String Theory

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories

QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Erlich et al

Chiral Symmetry Breaking in AdS/QCD

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted z^2 distribution, not constant condensate**

$$\delta M^2 = -2m_q < \bar{\psi}\psi > \times \int dz \ \phi^2(z) z^2$$

- **z^2 weighting consistent with higher Fock states at periphery of hadron wavefunction**

- **AdS/QCD: confined condensate**

- **"In-Hadron" Condensates**

de Teramond, Shrock, sjb

Quark and Gluon condensates reside within hadrons, not vacuum

- **Light Front Vacuum trivial up to zero modes**
- **Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs** — Casher and Susskind
- **Bound-State Bethe-Salpeter Equations** — Roberts et al.
- **Analogous to finite size superconductor**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict** — Shrock and sjb
- **"In-Hadron" Condensates**

- Color Confinement: Maximum Wavelength of Quark and Gluons
- Conformal symmetry of QCD coupling in IR
- Provides Conformal Template
- Motivation for AdS/QCD
- QCD Condensates inside of hadronic LFWFs
- Technicolor: confined condensates inside of technihadrons -- alternative to Higgs
- Simple physical solution to cosmological constant conflict with Standard Model

Shrock and sjb